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Supply Chain repairable Model with the Multi-suppliers and Single Demander

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Abstract This paper studies supply chain repairable system with a number of the same type of independent parts suppliers and the single demander. Suppose that the supply time of the component suppliers has exponential distribution, and the repair/restore time after the supply break has general continuous distribution. Using supplementary variable method, generalized Markov process theory and the Laplace transform technique, the important reliability indices of system are obtained. These indices have significant meaning for evaluating and measuring the supply chain system performance and improving the management level of the supply chain system.

Keywords Supply chain; Supplementary variable method; Generalized Markov process; Availability; Failure frequency

1 Introduction

Supply chain is defined as the overall network function structural model which is around the core enterprise, and this structure include raw materials' procurement, intermediate products' making, and the final products' selling. Suppliers, manufacturers, vendors, and the final users form a network structure (Figure 1). It runs through the supply chain that logistics flow, and capital flow, and information flow (see Ref. [1]). Supply chain reliability is defined as the probability of the chain meeting mission requirements to provide the required supplies to the critical transfer points within the system (see Ref. [2]).

Thomas was the first researcher that proposed reliability engineering would be applied in the supply chain, and gave the definition of the reliability of the supply chain in 2002(see Ref. [2]). To achieve supply chain reliability, a number of researchers have devoted their efforts in developing models to describe the elements and activities of a supply chain. There are some models such as multi-stage supply chain model (see Ref. [3]), economic model (see Ref. [4]), simulation model (see Ref. [5]) and stochastic model (see Ref. [6]). Moreover, Wang and Zhang analyzed the reliability of the multi-stage supply chain based on the Markov process in 2003(see Ref. [7]). Chen et al. analyzed the reliability and the network models of supply chain in 2004(see Ref. [8]). In order to simplify the calculation, Mu and Du proposed the concept of basic unit on the basis of different stages of the supply chain in the same year (see Ref. [9]). Zeng and Li researched the

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reliability problems of the basic structure model of the supply chain by using analytic hierarchy process in 2005(see Ref. [10]). Du and Mu established the mathematical model which is about overall reliability of the supply chain, the scale of soft-Union and the selection ways of enterprise in 2006(see Ref. [11]). Cai and Zeng performed quantitative analysis of the supply chain reliability by using GO algorithm in 2007(see Ref. [1]). Liu et al. introduced k-out-n: G system into the supply chain system, and set up an new model of supply chain, and some important indices of the reliability were obtained in 2008(see Ref. [12]). However, in above models, the authors either considered that the system was non-repairable or assumed that all random variables have exponential distribution. Obviously, many of the actual situations are not the cases. In addition, in practical problems, a demander makes often the contract with several suppliers to reduce the risk, therefore, this is a significant object that research the problems of supply chain with the multi-suppliers and single demander.

This paper is organized as follows: The supply chain repairable model with multi-suppliers and a single demander is introduced in Section 2, and the state equations of supply system are set up and their solutions are obtained in Section 3. Section 4 gets the reliability indices. Finally, some concluding remarks are given in Section 5.

2 The Model with the Multi-suppliers and Single Demander

The model with the multi-suppliers and single demander is introduced here by making the following assumptions:

A1. Supply chain system consists of n parts suppliers and the single demander. The supply time X of each component supplier has exponential distribution $F(t)$, $F(t) = 1 - e^{-\lambda t}$, $\lambda > 0$, and the repair/restore time Y of each of the parts suppliers has general continuous distribution $G(t)$, $G(t) = 1 - e^{-\int_0^t \mu(x)dx}$ and $E(Y) = \frac{1}{\mu}$.

A2. n parts suppliers can normally supply parts when $t = 0$ .There is a administrator in the supply chain system. If parts suppliers can not normally supply parts (known as failure), then they have been orderly optimized and restored by the administrator(known as maintenance). If the administrator is busy, the enterprises which can not normally supply parts wait for optimization and adjustment in line. The enterprises which are adjusted and restored are the same as the enterprises which can normally supply parts (as good as new after the repair), and immediately transferred to the operational state. Once there are the k parts suppliers supply fail in the system, then the system fails (that is, demander can not normally produce). When the supply chain system fails, it is closed, and the enterprises which have not failed will not fail.

A3. All the supply time of the component suppliers and the time of optimization and adjustment are independent mutually.

To analyze the reliability of the system, first of all, we define the state of system:

Let $X(t)$ be the number of parts suppliers which fail at time t , then $X(t) = i$, $i = 0,1,2,\cdots,k$, where $X(t) = k$ is only failure state of the system. Notice that $\{X(t), t \geq 0\}$ can not form a Markov process since the repair/restore time of the parts suppliers has general continuous distribution. Now we construct a generalized Markov process by introducing a supplementary variable. Let $Y(t)$ denote the optimized and adjusted time of the parts supplier at time t when $X(t) = 1,2,\cdots,k$. From A1-A3, we have $(X(t),Y(t))$ is a generalized a continuous-time homogeneous Markov process. That is, at any time t , when the specific values of $X(t)$ and $Y(t)$ are given, the process $(X(t),Y(t))$ after time t is independent of this process' history before time t. By the model assumptions, we can see the possible state transition (see Figure 2), where $\lambda_i = (n-i)\lambda, i = 0,1,\cdots,k-1$.

Furthermore, we give the following definition: $P_0(t) = P\{$ system stays on state 0 at time t $\}$, $P_i(t,x)dx = P\{$system stays on state i , and has remained for x time, then leaves from state i in $(x,\ x+dx)\}$, $i = 1,2,\cdots,k$.

3 The State Equations of Supply Chain System and their Solutions

Before state our main results, we introduce some more notation.

$$\bar{G}(x) = 1 - G(x)$$

$$P_i^*(s,x) = \int_0^\infty P_i(t,x)e^{-st}dt, i = 1,2,\cdots,k,$$

$$h_{i,j} = \int_0^\infty \mu(x)\bar{G}(x)e^{-(s+i\lambda)x}(1-e^{-\lambda x})^j dx$$

$$H_{i,j} = \int_0^\infty \bar{G}(x)e^{-(s+i\lambda)x}(1-e^{-\lambda x})^j dx,$$

$$a_1(s) = \frac{1}{h_{n-1,0}(s)}, b_1(s) = 0$$

$$a_2(s) = -\frac{(n-1)h_{n-2,1}(s)-1}{h_{n-1,0}(s)h_{n-2,0}(s)}, b_2(s) = \frac{-1}{h_{n-2,0}(s)},$$

$$a_i(s) = \sum_{j=1}^{i-1} \frac{-\binom{n-j}{i-j}h_{n-i,i-j}(s)a_j(s)}{h_{n-i,0}(s)} + \frac{a_{i-1}(s)}{h_{n-i,0}(s)}, i = 3,4,\cdots,k-1.$$

$$b_i(s) = \sum_{j=2}^{i-1} \frac{-\binom{n-j}{i-j}h_{n-i,i-j}(s)b_j(s)}{h_{n-i,0}(s)} + \frac{b_{i-1}(s)}{h_{n-i,0}(s)}, i = 3,4,\cdots,k-1.$$

According to Section 2, we get the Kolmogorow forward equations are as follows:

$$(\frac{d}{dt} + n\lambda)P_0(t) = \int_0^\infty P_1(t,x)\mu(x)dx, \tag{1}$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + (n-1)\lambda + \mu(x))P_1(t,x) = 0, \tag{2}$$

$$(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+(n-i)\lambda+\mu(x))P_i(t,x)=(n-i+1)\lambda P_{i-1}(t,x), \tag{3}$$

$$i=2,3,\cdots,k-1,$$

$$(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu(x))P_k(t,x)=(n-k+1)\lambda P_{k-1}(t,x). \tag{4}$$

Boundary conditions:

$$P_1(t,0)=n\lambda P_0(t)+\int_0^\infty P_2(t,x)\mu(x)dx, \tag{5}$$

$$P_i(t,0)=\int_0^\infty P_{i+1}(t,x)\mu(x)dx, i=2,3,\cdots,k-1, \tag{6}$$

$$P_k(t,0)=0. \tag{7}$$

And the initial conditions:

$$P_0(0)=1, P_i(0,x)=0, i=1,2,\cdots,k. \tag{8}$$

Now, by taking Laplace transform on both sides of (1)- (7), with the help of (8), we have

$$(s+n\lambda)P_0^*(s)-1=\int_0^\infty P_1^*(s,x)\mu(x)dx, \tag{9}$$

$$\frac{d}{dx}P_1^*(s,x)+(s+(n-1)\lambda+\mu(x))P_1^*(s,x)=0, \tag{10}$$

$$\frac{d}{dx}P_i^*(s,x)+(s+(n-i)\lambda+\mu(x))P_i^*(s,x)=(n-i+1)\lambda P_{i-1}^*(s,x), \tag{11}$$

$$i=2,3,\cdots,k-1,$$

$$\frac{d}{dx}P_k^*(s,x)+(s+\mu(x))P_k^*(s,x)=(n-k+1)\lambda P_{k-1}^*(s,x). \tag{12}$$

$$P_1^*(s,0)=n\lambda P_0^*(s)+\int_0^\infty P_2^*(s,x)\mu(x)dx, \tag{13}$$

$$P_i^*(s,0)=\int_0^\infty P_{i+1}^*(s,x)\mu(x)dx, i=2,3,\cdots,k-1, \tag{14}$$

$$P_k^*(s,0)=0. \tag{15}$$

Solving the differential equation (10), we get

$$P_1^*(s,x)=P_1^*(s,0)e^{-[s+(n-1)\lambda]x}\bar{G}(x). \tag{16}$$

From (16) and (11), and using the mathematical induction method, we have

$$P_i^*(s,x)=\sum_{j=1}^{i}\binom{n-j}{i-j}P_j^*(s,0)e^{-[s+(n-i)\lambda]x}(1-e^{-\lambda x})^{i-j}\bar{G}(x), \tag{17}$$

$$i=2,3,\cdots,k-1.$$

Thus, solving the differential equation (12) , and with the help of (17), we obtain

$$P_k^*(s,x) = \sum_{j=1}^{k-1} P_j^*(s,0) e^{-sx} \bar{G}(x) - \sum_{j=1}^{k-1} P_j^*(s,x), \tag{18}$$

Furthermore, we obtain (see appendix)

$$P_i^*(s,0) = [a_i(s)s + (a_i(s) + b_i(s))n\lambda] P_0^*(s) - a_i(s), \tag{19}$$

$$i = 1, 2, \cdots, k-1.$$

where

$$P_0^*(s) = \frac{1 + (1 - h_{0,0}(s)) \sum\limits_{j=1}^{k-1} a_j(s)}{s + (1 - h_{0,0}(s)) \sum\limits_{j=1}^{k-1} [a_j(s)s + (a_j(s) + b_j(s))n\lambda]}. \tag{20}$$

4 Reliability Indices of Supply Chain System

4.1 Availability

Let $A(t)$ denote instantaneous availability of supply chain system at time t, from formula of the availability(see Ref. [13]), we have $A(t) = P_0(t) + \sum_{i=1}^{k-1} \int_0^\infty P_i(t,x)dx$.

Taking the Laplace transforms on both sides of $A(t)$, and by using (17) and (19), we have

$$A^*(s) = P_0^*(s) + \sum_{i=1}^{k-1} \int_0^\infty P_i^*(s,x)dx = P_0^*(s) + \sum_{i=1}^{k-1} \sum_{j=1}^{i} H_{n-i,i-j}(s) \binom{n-j}{i-j} P_j^*(s,0)$$

$$= [1 + \sum_{i=1}^{k-1} \sum_{j=1}^{i} H_{n-i,i-j}(s) \binom{n-j}{i-j} (a_j(s)s + (a_j(s) + b_j(s))n\lambda)] P_0^*(s)$$

$$- \sum_{i=1}^{k-1} \sum_{j=1}^{i} H_{n-i,i-j}(s) \binom{n-j}{i-j} a_j(s).$$

Since $\lim\limits_{s\to 0} h_{0,0}(s) = 1$ and $\lim\limits_{s\to 0} \frac{1-h_{0,0}(s)}{s} = E(Y) = \frac{1}{\mu}$, thus, from (20),we have $\lim\limits_{s\to 0} sP_0^*(s) = \frac{1}{1+\frac{n\lambda}{\mu}\sum_{i=1}^{k-1}[a_i(0)+b_i(0)]}$. Therefore, the stationary availability of supply chain system is given by

$$A = \lim_{s\to 0} sA^*(s) = \frac{1 + n\lambda \sum\limits_{i=1}^{k-1} \sum\limits_{j=1}^{i} H_{n-i,i-j}(0) \binom{n-j}{i-j} (a_j(0) + b_j(0))}{1 + \frac{n\lambda}{\mu} \sum\limits_{i=1}^{k-1} (a_i(0) + b_i(0))}. \tag{21}$$

4.2 Failure Frequency

Let $M_f(t)$ denote the expected failure number of the system during interval $[0,t]$, then $M_f(t) = \int_0^t W_f(x)dx$, where $W_f(t)$ is the failure frequency at time t. From formula of the failure frequency (see Ref. [14]), we get $W_f(t) = \int_0^\infty (n-k+1)\lambda P_{k-1}(t,x)dx$. Thus, by taking the Laplace transforms on both sides of $W_f(t)$, with the help of (17) and (19) , we have

$$W_f^*(s) = (n-k+1)\lambda[\sum_{i=1}^{k-1} H_{n-k+1,k-1-i}(s)\binom{n-i}{k-1-i}(a_i(s)s+(a_i(s)+b_i(s))n\lambda)P_0^*(s)$$

$$-\sum_{i=1}^{k-1} H_{n-k+1,k-1-i}(s)\binom{n-i}{k-1-i}a_i(s)].$$

Therefore, the stationary failure frequency of supply chain system is given by

$$f = \lim_{t\to\infty}\frac{M_f(t)}{t} = \lim_{s\to 0} sW_f^*(s) = \frac{n(n-k+1)\lambda^2\sum_{i=1}^{k-1} H_{n-k+1,k-1-i}(0)\binom{n-i}{k-1-i}(a_i(0)+b_i(0))}{1+\frac{n\lambda}{\mu}\sum_{i=1}^{k-1}(a_i(0)+b_i(0))}. \quad (22)$$

5 Conclusion

In this work, we study a repairable model about supply chain system with the Multi-suppliers and single demander. By using supplementary variable method, generalized Markov process theory and the Laplace transform technique, we obtain the some reliability indices of the system. Our research is a useful exploration for the theoretical study or the prac

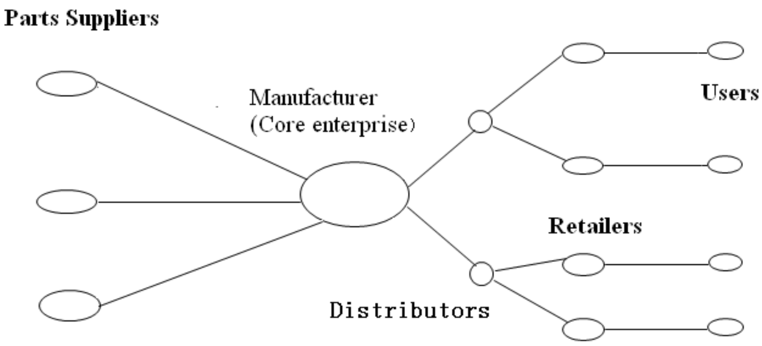

Figure 1: The pattern of network chain

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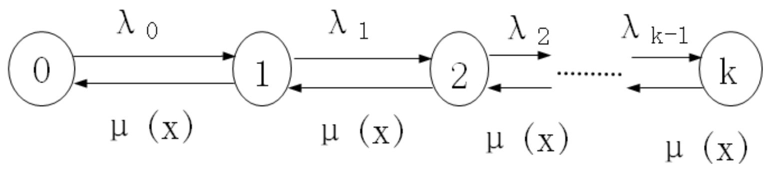

Figure 2: The state transition diagram for the supply chain system

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Appendix

The proof of (19)and (20):

From (9) and (16), we obtain

$$(s+n\lambda)P_0^*(s)-h_{n-1,0}(s)P_1^*(s,0)=1, \tag{23}$$

by (13) and (17), we have

$$n\lambda P_0^*(s) + [(n-1)h_{n-2,1}(s) - 1]P_1^*(s,0) + h_{n-2,0}(s)P_2^*(s,0) = 0, \tag{24}$$

From (14) and (17), we get

$$\sum_{j=1}^{i-1} \binom{n-j}{i-j+1} h_{n-i-1,i-j+1}(s)P_j^*(s,0) + [(n-i)h_{n-i-1,1}(s) - 1]P_i^*(s,0) + h_{n-i-1,0}(s)P_{i+1}^*(s,0) = 0, \tag{25}$$

$$i = 2,3,\cdots,k-2,$$

from (14), (18), (16), and (13), we obtain

$$P_{k-1}^*(s,0) = \sum_{j=1}^{k-1} h_{0,0}(s)P_j^*(s,0) - \sum_{j=1}^{k-2} P_j^*(s,0) - h_{n-1,0}(s)P_1^*(s,0) + n\lambda P_0^*(s), \tag{26}$$

substituting (23) into (26), we have

$$sP_0^*(s) + [1 - h_{0,0}(s)] \sum_{j=1}^{k-1} P_j^*(s,0) = 1, \tag{27}$$

thus, from (23), we have

$$P_1^*(s,0) = \frac{(s+n\lambda)P_0^*(s)}{h_{n-1,0}(s)} - \frac{1}{h_{n-1,0}(s)} = [a_1(s)s + a_1(s)n\lambda]P_0^*(s) - a_1(s), \tag{28}$$

From (24) and (28), we have

$$P_2^*(s,0) = [a_2(s)s + (a_2(s)+b_2(s))n\lambda]P_0^*(s) - a_2(s), \tag{29}$$

substituting (28) and (29) into (25), we get

$$P_i^*(s,0) = [a_i(s)s + (a_i(s)+b_i(s))n\lambda]P_0^*(s) - a_i(s), \tag{30}$$

$$i = 3,4,\cdots,k-1.$$

Substituting (28), (29) and (30) into (27), we get

$$P_0^*(s) = \frac{1 + (1-h_{0,0}(s)) \sum_{j=1}^{k-1} a_j(s)}{s + (1-h_{0,0}(s)) \sum_{j=1}^{k-1} [a_j(s)s + (a_j(s)+b_j(s))n\lambda]}. \tag{31}$$