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A Multi-Item Inventory Control Model for Perishable Items with Two Shelves

Hiroyuki Wakinaga[1,*] Katsushige Sawaki[2,†]

[1]Graduate School of Mathematical Sciences and Information Engineering, Nanzan University, Aichi, Japan
[2]Graduate School of Business Administration, Nanzan University, Aichi, Japan

Abstract In this paper a multi-item inventory control model for perishable items which can be sold on the two shelves. The first shelf is for normal sales shelf and includes a number of single items. The second shelf is for sales promotion and consists of multiple items. We show that there exists an optimal policy so as to minimize the expected total cost for *N* items consisting of a general convex inventory or salvage and penalty cost functions. We also explore an optimal ordering policy of a finite horizon inventory model with a sales promotion shelf capacity constraint under the newsboy problem setting.

Keywords Inventory Control; Revenue Management; Perishable Items; Promotion Shelf

1 Introduction

In recent years, an important task of retailer is to set up the counters to suit customer preference. A merchandising method or planogram which efficiently allocates items is required. In particular, sales promotion items or perishable items must be allocated in noticeable sales zones. For example, items for sales promotion are often displayed in front of the cash register. When a customer enters the store, he/she can be influenced to buy more than he/she originally intended. In other words, retailers should control their inventory level by allocating store items in consideration of the purchasing behavior of the customer.

Models with multiple items have been written by different approaches for treating the various aspects. Cachon[1] discussed an inventory model for multiple items that minimizes the expected total cost with shelves, inventory and transportation costs. Lee and Hersh[4] developed a discrete time dynamic programming model for airline seat inventory control with a nested seat allocation problem. In order to the improve revenues, airline seats to sell are allocated at different prices and booking classes. Masui[5] presented an M-item two-stage inventory model which maximizes the expected profit with storage constraints. Sharma[9] determined an optimal production quantity and production time for multiple items with a fixed use period using a classical EOQ model.

*Email: d04mm002@nanzan-u.ac.jp
†Email: sawaki@ms.nanzan-u.ac.jp

In this paper, with reference to Gallego and Moon[3], and Wakinaga and Sawaki[11], an inventory control model for perishable items which can be sold on two types of the shelves is considered. The first type of shelf is for normal sales and the second type of shelf is for sales promotion. Usually, the second shelf is located in a place which easily draws the more attension from the customers. By introducing such a shelf for the sales promotion, greater demand potential can be created based upon the price reduction or reallocation of perishable items.

This paper is organized as follows. In Section 2, we explain the assumptions of this model to be formulated, along with notations. In Section 3, we present the formulation including two shelves as a two periods model. Finally, in Section 4 results obtained in this paper are summarized and concluding remarks are presented.

2 Assumptions

We assume that a planning horizon consists of two periods and make a decision to order at the beginning of each period. Also no leadtime after ordering is assumed. Let shelf 1 be the shelf for normal sales and shelf 2 be the shelf for sales promotion. Next, the sequence of operations for two periods is explained. For any item, initial inventory level at period 1 is set at zero. Hence, in period 1, the ordering quantity for any item is for sales on the shelf 1 only. Any leftover stock at the end of period 1 is retained and can be provided for sales in the following period. In this regard, a positive inventory at the end of period 1 incurs a holding cost. When the inventory level is negative at the end of period 1, this backlog cannot be compensated by the ordering quantity of period 2. Thus a backlog at the end of period 1 incurs a penalty cost. In period 2, a more rate for shelf 2 is set to move inventory at the end of period 1 (Figure 1). When the inventory is moved from shelf 1 to shelf 2, setup and move costs occur. The inventory of shelf 1 is then comprised of any unmoved inventory at the end of period 1 and the inventory after ordering. In this model, when these inventories exist on the shelf, the inventories with a shorter sales period are sold first. Any remaining stock at the end of period 2 which is the last sales period incurs a salvage cost. However, a backlog of shelf 2 is without negative cost. A backlog of shelf 1 can be compensated by the leftover stock from shelf 2.

The following notations in an optimal ordering model with discrete times $i = 1,2$, and item $j = 1,2,\cdots,n$ are used in this model.

x_{ij}:the inventory level of item j observed before ordering at the begining of period i

y_{ij}:the inventory level of item j observed after ordering at the begining of period i decision variable $[y_j \geq x_j]$

$\hat{c}_j$:ordering cost of item j per unit

p_j:penalty cost of item j per unit $[p_j > \hat{c}_j]$

h_j:holding cost of item j per unit

s_j:salvage cost of item j per unit $[h_j > s_j]$

q:move cost per unit

K:setup cost for shelf 2

S:capacity of shelf 2

R:space capacity of shelf 2

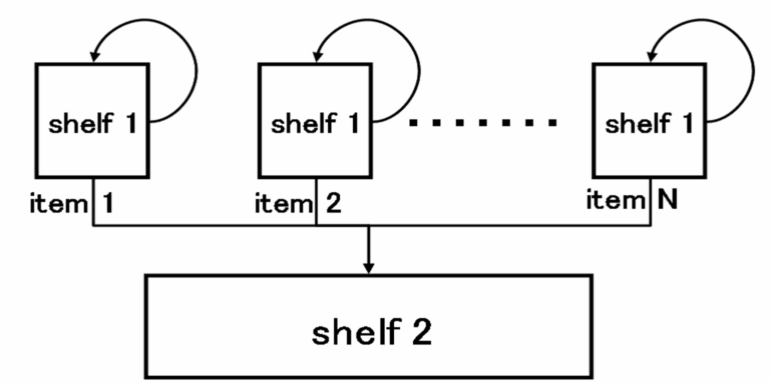

Figure 1: inventory move

β_j:the move rate of item j
V_{ij}:the demand of item j on shelf 1 in period i random variable
W_{ij}:the demand of item j on shelf 2 in period i random variable
$\psi_{ij}(v_{ij})$:the probability density function of V_{ij}
$\Psi_{ij}(v_{ij})$:the distribution function of V_{ij}
$\phi_{ij}(w_{ij})$:the probability density function of W_{ij}
$\Phi_{ij}(w_{ij})$:the distribution function of W_{ij}
C_{ij}:the total cost of item j in period i
TC_{ij}:the expected total cost of item j in period i

The demand for all periods of each shelf is non-negative and independent. Capacity of shelf 1 is large enough to hold the items. Since the capacity of shelf 2 is finite, the sales policy must change according to the relationship between move quantity of any item and space capacity of shelf 2. In this model, items are moved in relation to their priority with item 1. Items which cannot be moved to shelf 2 must be allocated to shelf 1. Therefore, the expected total cost for all items depends on a move rate which adjusts the number of items on a shelf 2.

3 Formulation

In this section, we discuss an optimal policy for two periods. First, the one period model for period 2 is considered. Next, the two periods model is considered.

3.1 Optimal Ordering Policy of Period 2

For any item j, the inventory level before ordering in period 2 is

$$x_{2j} = y_{1j} - V_{1j}. \tag{1}$$

Move quantity for shelf 2 consists of five patterns depending on the relationship between the space capacity of shelf 2 and the inventory at the begining of period 2.

- When space capacity of shelf 2 is positive

Pattern a $x_{2j} = 0$
Pattern b $R \geq \beta_j x_{2j}$
Pattern c $R < \beta_j x_{2j}$

- When space capacity of shelf 2 is zero

Pattern d $x_{2j} = 0$
Pattern e $x_{2j} > 0$

An optimal policy for period 2 can be chracterized by these patterns.

For Pattern a. All the inventory of item j after ordering is sold on shelf 1 because there is no move inventory of item j. The total cost can be given as follows:

$$C_{2ja}(x_{2j}, y_{2ja}) = (y_{2ja} - x_{2j})\hat{c}_j + (y_{2ja} - V_{2j})^+ h_j + (V_{2j} - y_{2ja})^+ p_j. \tag{2}$$

Let $TC_{2ja}(x_{2j}, y_{2ja}) = E\left[C_{2ja}(x_{2j}, y_{2ja})\right]$ be the expected total cost. Thus

$$\begin{aligned} TC_{2ja}(x_{2j}, y_{2ja}) &= (y_{2ja} - x_{2j})\hat{c}_j \\ &+ h_j \int_0^{y_{2ja}} (y_{2ja} - v_{2j}) d\Psi_{2j}(v_{2j}) + p_j \int_{y_{2ja}}^{\infty} (v_{2j} - y_{2ja}) d\Psi_{2j}(v_{2j}). \end{aligned} \tag{3}$$

To find the minimizer, differentiation of $TC_{2ja}(x_{2j}, y_{2ja})$ with respect to y_{2ja} is taken. From the derivative we have

$$\frac{\partial}{\partial y_{2ja}} TC_{2ja}(x_{2j}, y_{2ja}) = \hat{c}_j - p_j + (h_j + p_j)\Psi_{2j}(y_{2ja}). \tag{4}$$

Putting $\frac{\partial}{\partial y_{2ja}} TC_{2ja}(x_{2j}, y_{2ja}) = 0$, in Pattern a, the optimal inventory level y_{2ja} after ordering in period 2 satisfies

$$\Psi_{2j}(y_{2ja}) = \frac{p_j - \hat{c}_j}{h_j + p_j}. \tag{5}$$

Both sides are positive by definition of the notations. The sufficient condition for a minimum of $TC_{2ja}(x_{2j}, y_{2ja})$ is $\frac{\partial^2}{\partial y_{2ja}^2} TC_{2ja}(x_{2j}, y_{2ja}) > 0$. Therefore we have the second derivative

$$\frac{\partial^2}{\partial y_{2ja}^2} TC_{2ja}(x_{2j}, y_{2ja}) = (h_j + p_j)\psi_{2j}(y_{ja}) > 0, \tag{6}$$

$TC_{2ja}(x_{2j}, y_{2ja})$ is clearly a convex function with respect to y_{2ja}.

For Pattern b. Since space capacity of shelf 2 is greater than or equal to the move quantity of item j, the total move quantity at the beginning of period 2 can be allocated on shelf 2. Hence, the inventory of item j after ordering is sold on both shelf 1 and shelf 2. The total cost can be given as follows:

$$\begin{aligned} C_{2jb}(x_{2j}, y_{2jb}) &= (y_{2jb} - x_{2j})\hat{c}_j + \beta x_{2j} q + K \\ &+ (y_{2jb} - V_{2j} + \beta_j x_{2j})^+ h_j + \left[(V_{2j} + \beta_j x_{2j} - y_{2jb})^+ - (\beta_j x_{2j} - W_{2j})^+\right]^+ p_j \\ &+ \left[\left\{(\beta_j x_{2j} - W_{2n})^+ - (V_{2n} + \beta_j x_{2j} - y_{2jb})^+\right\}^+ + \left\{(1 - \beta_j)x_{2j} - V_{2n}\right\}^+\right]^+ s_j. \end{aligned} \tag{7}$$

Let $TC_{2jb}(x_{2j},y_{2jb}) = E\left[C_{2jb}(x_{2j},y_{2jb})\right]$ be the expected total cost. Thus

$$\begin{aligned}
&TC_{2jb}(x_{2j},y_{2jb}) = (y_{2jb}-x_{2j})\hat{c}_j + \beta_j x_{2j} q + K \\
&+h_j \int_0^{(1-\beta_j)x_{2j}} (y_{2jb}-x_{2j}) d\Psi_{2j}(v_{2j}) + h_j \int_{(1-\beta_j)x_{2j}}^{y_{2jb}-\beta_j x_{2j}} (y_{2jb}-\beta_j x_{2j}-v_{2j}) d\Psi_{2j}(v_{2j}) \\
&+s_j \int_0^{(1-\beta_j)x_{2j}} (x_{2j}-\beta_j x_{2j}-v_{2j}) d\Psi_{2j}(v_{2j}) \\
&+s_j \left[\int_0^{y_{2jb}-\beta_j x_{2j}} \int_0^{\beta_j x_{2j}} (\beta_j x_{2j}-w_{2j}) d\Phi_{2j}(w_{2j}) d\Psi_{2j}(v_{2j}) \right. \\
&\left. + \int_{y_{2jb}-\beta_j x_{2j}}^{y_{2jb}} \int_0^{y_{2jb}-v_{2j}} (y_{2jb}-v_{2j}-w_{2j}) d\Phi_{2j}(w_{2j}) d\Psi_{2j}(v_{2j}) \right] \\
&+p_j \left[\int_0^{\beta_j x_{2j}} \int_{y_{2jb}-w_{2j}}^{\infty} (v_{2j}+w_{2j}-y_{2jb}) d\Psi_{2j}(v_{2j}) d\Phi_{2j}(w_{2j}) \right. \\
&\left. + \int_{\beta_j x_{2j}}^{\infty} \int_{y_{2jb}-\beta_j x_{2j}}^{\infty} (v_{2j}-y_{2jb}+\beta_j x_{2j}) d\Psi_{2j}(v_{2j}) d\Phi_{2j}(w_{2j}) \right]. \qquad (8)
\end{aligned}$$

To find the minimizer, differentiation of $TC_{2jb}(x_{2j},y_{2jb})$ with respect to y_{2jb} is taken. From the derivative we have

$$\begin{aligned}
\frac{\partial}{\partial y_{2jb}} TC_{2b}(x_{2j},y_{2jb}) &= \hat{c}_j - p_j + (h_j+p_j)\Psi_{2j}(y_{2jb}-\beta_j x_{2j}) \\
&+(s_j+p_j)\int_{y_{2jb}-\beta_j x_{2j}}^{y_{2jb}} \Phi_{2j}(y_{2jb}-v_{2j})\psi_{2j}(v_{2j})dv_{2j}. \qquad (9)
\end{aligned}$$

Putting $\frac{\partial}{\partial y_{2jb}} TC_{2jb}(x_{2j},y_{2jb}) = 0$, in Pattern b, the optimal inventory level y_{2jb} after ordering in period 2 satisfies

$$\begin{aligned}
&(h_j+p_j)\Psi_{2j}(y_{2jb}-\beta_j x_{2j}) + (s_j+p_j)\int_{y_{2jb}-\beta_j x_{2j}}^{y_{2jb}} \Phi_{2j}(y_{2jb}-v_{2j})\psi_{2j}(v_{2j})dv_{2j} \\
&= p_j - \hat{c}_j. \qquad (10)
\end{aligned}$$

Both sides are positive by definition of the notations. The sufficient condition for a minimum of $TC_{2jb}(x_{2j},y_{2jb})$ is $\frac{\partial^2}{\partial y_{2jb}^2} TC_{2jb}(x_{2j},y_{2jb}) > 0$. Therefore we have the second derivative

$$\begin{aligned}
&\frac{\partial^2}{\partial y_{2jb}^2} TC_{2jb}(x_{2j},y_{2jb}) = (h_j+p_j)\psi_{2j}(y_{2jb}-\beta_j x_{2j}) \\
&+(s_j+p_j)\left[\int_{y_{2jb}-\beta_j x_{2j}}^{y_{2jb}} \phi_{2j}(y_{2jb}-v_{2j})\psi_{2j}(v_{2j})dv_{2j} - \Phi_{2j}(\beta_j x_{2j})\psi_{2j}(y_{2jb}-\beta_j x_{2j})\right]. \qquad (11)
\end{aligned}$$

By $h_j > s_j$, we obtain $(h_j+p_j)\psi_{2j}(y_{2jb}-\beta_j x_{2j}) > (s_j+p_j)\Phi_{2j}(\beta_j x_{2j})\psi_{2j}(y_{2jb}-\beta_j x_{2j})$; thus $TC_{2jb}(x_{2j},y_{2jb})$ is convex function with respect to y_{2jb}.

For Pattern c. Since space capacity of shelf 2 is less than the move quantity fo item j, an equal quantity with space capacity can be allocated on the shelf 2. Hence, the inventory of item j after ordering is sold on both shelf 1 and shelf 2. The total cost can be given as follows:

$$\begin{aligned} C_{2jc}(x_{2j}, y_{2jc}) &= (y_{2jc} - x_{2j})\hat{c}_j + Rq + K \\ &+ (y_{2jc} - V_{2j} + R)^+ h_j + \left[(V_{2j} + R - y_{2jc})^+ - (R - W_{2j})^+\right]^+ p_j \\ &+ \left[\left\{(R - W_{2n})^+ - (V_{2n} + R - y_{2jc})^+\right\}^+ + \left\{x_{2j} - R - V_{2j}\right\}^+\right]^+ s_j, \end{aligned} \tag{12}$$

where $R = S - \beta_j \sum_{n=1}^{j-1} x_{2n}$. Let $TC_{2jc}(x_{2j}, y_{2Jc}) = E\left[C_{2jc}(x_{2j}, y_{2jc})\right]$ be the expected total cost. Thus

$$\begin{aligned} TC_{2jc}(x_{2j}, y_{2jc}) &= (y_{2jc} - x_{2j})\hat{c}_j + Rq + K + s_j \int_0^{x_{2j}-R} (x_{2j} - R - v_{2j}) d\Psi_{2j}(v_{2j}) \\ &+ h_j \int_0^{x_{2j}-R} (y_{2jc} - x_{2j}) d\Psi_{2j}(v_{2j}) + h_j \int_{x_{2j}-R}^{y_{2jb}-R} (y_{2jb} - R - v_{2j}) d\Psi_{2j}(v_{2j}) \\ &+ s_j \left[\int_0^{y_{2jc}-R} \int_0^R (R - w_{2j}) d\Phi_{2j}(w_{2j}) d\Psi_{2j}(v_{2j}) \right. \\ &+ \left. \int_{y_{2jc}-R}^{y_{2jc}} \int_0^{y_{2jc}-v_{2j}} (y_{2jb} - v_{2j} - w_{2j}) d\Phi_{2j}(w_{2j}) d\Psi_{2j}(v_{2j}) \right] \\ &+ p_j \left[\int_0^R \int_{y_{2jc}-w_{2j}}^{\infty} (v_{2j} + w_{2j} - y_{2jc}) d\Psi_{2j}(v_{2j}) d\Phi_{2j}(w_{2j}) \right. \\ &+ \left. \int_R^{\infty} \int_{y_{2jb}-R}^{\infty} (v_{2j} - y_{2jb} + R) d\Psi_{2j}(v_{2j}) d\Phi_{2j}(w_{2j}) \right]. \end{aligned} \tag{13}$$

To find the minimizer, differentiation of $TC_{2jc}(x_{2j}, y_{2jc})$ with respect to y_{2jc} is taken. From the derivative we have

$$\begin{aligned} \frac{\partial}{\partial y_{2jc}} TC_{2c}(x_{2j}, y_{2jc}) &= \hat{c}_j - p_j + (h_j + p_j)\Psi_{2j}(y_{2jc} - R) \\ &+ (s_j + p_j) \int_{y_{2jc}-R}^{y_{2jc}} \Phi_{2j}(y_{2jc} - v_{2j}) \psi_{2j}(v_{2j}) dv_{2j}. \end{aligned} \tag{14}$$

Putting $\frac{\partial}{\partial y_{2jc}} TC_{2jc}(x_{2j}, y_{2jc}) = 0$, in Pattern c, the optimal inventory level y_{2jc} after ordering in period 2 satisfies

$$\begin{aligned} &(h_j + p_j)\Psi_{2j}(y_{2jc} - R) + (s_j + p_j) \int_{y_{2jc}-R}^{y_{2jc}} \Phi_{2j}(y_{2jc} - v_{2j}) \psi_{2j}(v_{2j}) dv_{2j} \\ &= p_j - \hat{c}_j. \end{aligned} \tag{15}$$

Both sides are positive by definition of the notations. The sufficient condition for a minimum of $TC_{2jc}(x_{2j}, y_{2jc})$ is $\frac{\partial^2}{\partial y_{2jc}^2} TC_{2jc}(x_{2j}, y_{2jc}) > 0$. Therefore we have the second derivative

$$\begin{aligned} &\frac{\partial^2}{\partial y_{2jc}^2} TC_{2jc}(x_{2j}, y_{2jc}) = (h_j + p_j)\psi_{2j}(y_{2jc} - R) \\ &+ (s_j + p_j) \left[\int_{y_{2jc}-R}^{y_{2jc}} \phi_{2j}(y_{2jc} - v_{2j}) \psi_{2j}(v_{2j}) dv_{2j} - \Phi_{2j}(R) \psi_{2j}(y_{2jc} - R)\right]. \end{aligned} \tag{16}$$

By $h_j > s_j$, we obtain $(h_j + p_j)\psi_{2j}(y_{2jc}) - R > (s_j + p_j)\Phi_{2j}(R)\psi_{2j}(y_{2jc} - R)$;
thus $TC_{2jc}(x_{2j}, y_{2jc})$ is convex function with respect to y_{2jc}.

For Pattern d. Since there is no inventory at the begining of period 2, all the inventory of item j after ordering is sold on shelf 1 even if there is space capacity of shelf 2. The total cost is the same as equation (2). The following sequences also are the same as Pattern a.

For Pattern e. Although there is an inevntory at the begining of period 2, this inventory can not be allocated on shelf 2 because there is no space capacity of shelf 2. Hence, all the inventory of item j after ordering is sold on shelf 1. The total cost can be given as follows:

$$\begin{aligned} C_{2je}(x_{2j}, y_{2je}) = (y_{2je} - x_{2j})\hat{c}_j + (y_{2je} - V_{2j})^+ h_j \\ +(V_{2j} - y_{2je})^+ p_j + (x_{2j} - V_{2j})^+ s_j. \end{aligned} \tag{17}$$

Let $TC_{2je}(x_{2j}, y_{2Je}) = E\left[C_{2je}(x_{2j}, y_{2je})\right]$ be the expected total cost. Thus

$$\begin{aligned} TC_{2jc}(x_{2j}, y_{2jc}) = (y_{2jc} - x_{2j})\hat{c}_j + \int_0^{x_{2j}} \left\{(x_{2j} - v_{2j})s_j + (y_{2je} - x_{2j})h_j\right\} d\Psi_{2j}(v_{2j}) \\ +h_j \int_{x_{2j}}^{y_{2je}} (y_{2je} - v_{2j}) d\Psi_{2j}(v_{2j}) + p_j \int_{y_{2je}}^{\infty} (v_{2j} - y_{2je}) d\Psi_{2j}(v_{2j}) \end{aligned} \tag{18}$$

To find the minimizer, differentiation of $TC_{2je}(x_{2j}, y_{2je})$ with respect to y_{2je} is taken. The first derivative is the same as equation (4). The following sequences also are also the same as Pattern a.

3.2 Optimal Ordering Policies for Two Periods

The expected total cost after period 1 is an optimal policy over the two periods, and includes the expected total costs of period 2 with five patterns. For any item j, when we set $x_{1j} = 0$, the expected total cost after period 1 is

$$\begin{aligned} & TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je}) = (y_{1j} - x_{1j})\hat{c}_j \\ & +L(x_{1j}, y_{1j}) + E\left[TC_{2ja}(y_{1j} - v_{1j}, y^*_{2ja})\right. \\ & \left.+TC_{2jb}(y_{1j} - v_{1j}, y^*_{2jb}) + TC_{2jc}(y_{1j} - v_{1j}, y^*_{2jc})\right]\delta(R) \\ & +E\left[TC_{2jd}(y_{1j} - v_{1j}, y^*_{2jd}) + TC_{2je}(y_{1j} - v_{1j}, y^*_{2je})\right]\delta(-R), \end{aligned} \tag{19}$$

where

$$\delta(r) = \begin{cases} 1 & \text{if } \ r > 0 \\ 0 & \text{otherwise} \end{cases}.$$

$L(x_{1j}, y_{1J})$ shows the expected one-period holding and penalty cost function. To find the minimizer of $TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je})$, differentiation of equation (19) with respect to y_{1j} can be written as follows:

$$\begin{aligned}
&\frac{\partial}{\partial y_{1j}} TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je}) = \hat{c}_j - p_j + (h_j + p_j)\Psi_{1j}(y_{1j}) \\
&+ \Bigg(\int_{y_{1j}}^{\infty} \frac{\partial}{\partial y_{1j}} TC_{2ja}(y_{1j} - v_{1j}, y^*_{2ja}) d\Psi_{1j}(v_{1j}) \\
&+ \int_{y_{1j}-R}^{y_{1j}} \frac{\partial}{\partial y_{1j}} TC_{2jb}(y_{1j} - v_{1j}, y^*_{2jb}) d\Psi_{1j}(v_{1j}) \\
&+ \int_{0}^{y_{1j}-R} \frac{\partial}{\partial y_{1j}} TC_{2jc}(y_{1j} - v_{1j}, y^*_{2jc}) d\Psi_{1j}(v_{1j}) \Bigg) \delta(R) \\
&+ \Bigg(\int_{y_{1j}}^{\infty} \frac{\partial}{\partial y_{1j}} TC_{2jd}(y_{1j} - v_{1j}, y^*_{2jd}) d\Psi_{1j}(v_{1j}) \\
&+ \int_{0}^{y_{1j}} \frac{\partial}{\partial y_{1j}} TC_{2je}(y_{1j} - v_{1j}, y^*_{2je}) d\Psi_{1j}(v_{1j}) \Bigg) \delta(-R) \\
&+ \Big\{ \Big(TC_{2jc}(R) - TC_{2jb}(R) \Big) \psi_{1j}(y_{1j} - R) \Big\} \delta(R), \qquad (20)
\end{aligned}$$

where

$$\frac{\partial}{\partial y_{1j}} TC_{2ja}(y_{1j} - v_{1j}, y^*_{2ja}) = -\hat{c}_j, \qquad (21)$$

$$\begin{aligned}
&\frac{\partial}{\partial y_{1j}} TC_{2jb}(y_{1j} - v_{1j}, y^*_{2jb}) = -\hat{c}_j + \beta_j(p_j + q) \\
&-\beta_j(p_j + h_j)\Big\{\Psi_{2j}\Big(y_2 jb - \beta_j(y_{1j} - v_{1j})\Big)\Big\} + \beta_j p_j \Big\{\Phi_{2j}\Big(\beta_j(y_{1j} - v_{1j})\Big)\Big\} \\
&- \big\{h_j(1 - \beta_j) + s_j\beta_j\big\}\Psi_{2j}\Big((1 - \beta_j)(y_{1j} - v_{1j})\Big) \\
&(s_j + p_j)\int_0^{y_2 jb - \beta_j(y_{1j} - v_{1j})} \beta_j \Phi_{2j}\Big(\beta_j(y_{1j} - v_{1j})\Big) \\
&+ s_j\Big\{(1 - \beta_j)(v_{1j} + x_{2j} - y_{1j})\psi_{2j}\Big((1 - \beta_j)(y_{1j} - v_{1j})\Big)\Big\}, \qquad (22)
\end{aligned}$$

$$\frac{\partial}{\partial y_{1j}} TC_{2jc}(y_{1j} - v_{1j}, y^*_{2jc}) = -\hat{c}_j + (s_j - h_j)\Psi_{2j}(y_{1j} - R - v_{1j}), \qquad (23)$$

$$\frac{\partial}{\partial y_{1j}} TC_{2jd}(y_{1j} - v_{1j}, y^*_{2jd}) = -\hat{c}_j, \qquad (24)$$

$$\frac{\partial}{\partial y_{1j}} TC_{2je}(y_{1j} - v_{1j}, y^*_{2je}) = -\hat{c}_j + (s_j - h_j)\Psi_{2j}(y_{1j} - v_{1j}). \qquad (25)$$

The optimal inventory level after ordering for two periods is y_{1j} with the necessary condition such that a minimum of $TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je})$ is $\frac{\partial}{\partial y_{1j}} TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je}) = 0$. Furthermore the sufficient condition for a minimum of $TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je})$ is $\frac{\partial^2}{\partial y^2_{1j}} TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je})$. If $TC_{1j}(x_{1j}, y_{1j}, y^*_{2ja}, \ldots, y^*_{2je})$ is convex function, there is a unique optimal solution. However, arriving at the sufficient condition under a general demand distribution is difficult. Therefore, parameters and demand functions must be set in order to illustrate a convex function.

Finally, since the expected total cost of item j for two periods can be shown, the total expected cost of N items can be given as follows:

$$TC_1(x_1,y_1) = \min\left\{\sum_{n=1}^{N} TC_{1n}(x_{1n},y_{1n},y^*_{2na},\ldots,y^*_{2ne})\right\}. \tag{26}$$

4 Concluding Remarks

In this paper, we show a multi-item inventory model for perishable items which can be sold on two types of shelves. Focusing on the managerial task of perishable items, we formulate a multi-item inventory model for two periods with sales procedures under the capacity constraint of shelf 2 and the move rate. In particular, shelf 2 consists of multiple items. But all the items are not always allocated on shelf 2 due to a capacity constraint. That is, the total expected cost for N items fluctuates according to the number of items which are allocated on shelf 2. This multi-item inventory model is considered applicable to actual inventory management.

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