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A New Model and Calculation of Available Transfer Capability With Wind Generation*

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Abstract With the development of wind generation, the calculation of available transfer capability (denoted as ATC) considering wind generation has very high application value. Based on the maximum function, this paper presents an ATC model. The characteristic of the new model is twofold. First, it considers wind turbines connected to power system and static security of power system simultaneously. Second, it is a system of semismooth equations. By using the smoothing strategy, a smoothing Newton method is adopted for solving the proposed new ATC model. Numerical simulation results of the IEEE 30-bus and 118-bus system show the new model and algorithm are feasible and effective. The impact of wind turbines connected to power system on ATC is analyzed.

1 Introduction

Wind generation technologies are presently more and more mature，and wind power is fastest growing and most promising one among all renewable energy. According to statistical data of World Wind Energy Association, worldwide installed capacity of all wind turbines reaches 121188 MW by the end of 2008, out of which 27261 MW were added in 2008. Wind energy continued its growth in 2008 at an increased rate of 29 % [1]. Along with increasing level of wind generation in power networks, there are some new research problems to be studied for the power systems.

In power systems, available transfer capability (ATC) is the transfer capacity remaining in the physical transmission network for further commercial activity, over and above already committed uses [2]. Electric utilities require posting information

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on ATC of transmission systems so that such vital information can help power marketers, sellers and buyers in planning, operation and reserving the transmission services. ATC also is useful information for the operator to understand how far stability limits are from the operating point and can also indicate the transfer capacity which can be increased without compromising system security and stability [3].

Many researches have been done on the model and calculation of ATC, but the research work has been seldom studied about the model and calculation of ATC considering wind turbines connected to power system. It is necessary to construct a rational ATC model and effectual calculation method for power systems containing with wind generation. Hence, this paper focuses on the model and solution method of ATC with wind generation.

Reference [4] proposed the improved PQ model and presented RX model for modeling of wind farms in load flow analysis, and applied the impedance containing slip to be the equivalent of wind turbines. Reference [5] established a calculation model of continuation power flow by using the detailed π -type equivalent circuit of asynchronous generator. Reference [6] constructed static security based smoothing ATC model by using the pointwise maximum function.

Along the work of [5] and [6], this paper first derives the power flow model including wind turbines where the slip is as a new state variable. Then, by integrating power flow model combining with wind turbines and conventional static security constraints, a new semismooth ATC model is constructed based on the so-called pointwise maximum function. By using the smoothing strategy, the corresponding smooth ATC model is set up, and the smoothing Newton method is adopted to solve the new ATC model. Numerical simulation results of the IEEE 30-bus and 118-bus system show the proposed model and algorithm are feasible and effective. Finally, the impact of wind turbines connected to power system on ATC is analysed.

This paper is organized as follows. Section 2 constructs power flow model including wind turbines. In section 3, the static security constrained ATC considering wind generation is proposed, which is a semi-smooth model. In section 4, a smoothing algorithm is presented for solving the new ATC model. Numerical results and conclusions are given in sections 5 and 6 respectively.

2 Power Flow Model Including Wind Turbines

2.1 The Mathematic Model of Wind Turbines

Asynchronous generators are generally applied in wind turbines, as shown in Figure 1.

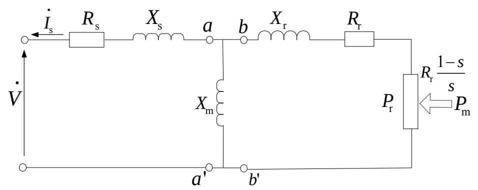

Figure 1: Equivalent circuit of asynchronous generator.

Where $R_s + jX_s$ is the stator impedance; $R_r + jX_r$ is the rotor impedance; X_m is the excitation reactance; s is the slip of the asynchronous generator; V is terminal voltage of wind turbine; P_r is the electric power on the rotor of asynchronous generators ; P_m is mechanical output of wind turbines, which can be obtained:

$$P_m = 0.5\rho A v^3 C_p \tag{1}$$

where ρ is the density of air; A is the rotor area; v is the wind speed; C_p is power coefficient.

After simplifying right circuit of aa' in Figure 1, Figure 2 can be gotten:

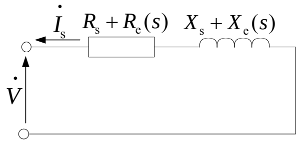

Figure 2: Equivalent circuit of asynchronous generator by simplifying right circuit of aa'.

Where $R_e(s) + jX_e(s)$ is equivalent impedance of right circuit of aa', as the following expressions:

$$R_e(s) = \frac{X_m^2 \frac{R_r}{s}}{(\frac{R_r}{s})^2 + (X_m + X_r)^2} \tag{2}$$

$$X_e(s) = \frac{X_m^2 X_r + X_m X_r^2 + X_m (\frac{R_r}{s})^2}{(\frac{R_r}{s})^2 + (X_m + X_r)^2} \tag{3}$$

Then power injecting into system can be obtained as:

$$P_e(V, s) = \frac{-[R_s + R_e(s)]V^2}{[R_s + R_e(s)]^2 + [X_s + X_e(s)]^2} \tag{4}$$

$$Q_e(V, s) = \frac{-[X_s + X_e(s)]V^2}{[R_s + R_e(s)]^2 + [X_s + X_e(s)]^2} \tag{5}$$

Using Thevenin's Theorem to simplify left circuit of bb' in Figure 1, Figure 3 can be gotten:

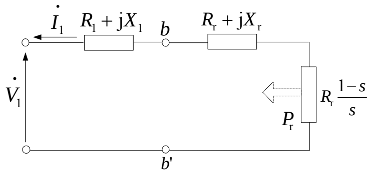

Figure 3: Thevenin's equivalent circuit of asynchronous generator.

Where $R_1 + \mathrm{j}X_1$ is the equivalent impedance, V_1 is the equivalent voltage, as following expressions:

$$R_1 = \frac{X_m^2 R_s}{R_s^2 + (X_s + X_m)^2} \tag{6}$$

$$X_1 = \frac{X_m R_s^2 + X_m X_s^2 + X_m^2 X_s}{R_s^2 + (X_s + X_m)^2} \tag{7}$$

$$V_1 = \frac{X_m}{\sqrt{R_s^2 + (X_s + X_m)^2}} V \tag{8}$$

The electric power on the rotor of generator can be calculated from the following equation:

$$P_r(V, s) = \frac{-V_1^2 R_r \frac{1-s}{s}}{(R_1 + \frac{R_r}{s})^2 + (X_1 + X_r)^2} \tag{9}$$

2.2 The Power Flow Model Including Wind Turbines

After wind turbines are connected to power system and s is introduced as new state variable, a group of balance equations should be added to original power flow equations. When the system reaches steady state, P_m and P_r should be equal according to power conservation principle.

Let i is the node connected with wind turbines, θ and V represent phase angle and voltage magnitude, then the power flow equations, corresponding to the node i, can be stated as:

$$\begin{cases} P_{ei}(V_i, s_i) - P_{Li} - V_i \sum_{j \in i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 \\ Q_{ei}(V_i, s_i) - Q_{Li} - V_i \sum_{j \in i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 \\ P_{mi} - P_{ri}(V_i, s_i) = 0 \end{cases} \tag{10}$$

3 ATC Model With Wind Generation

3.1 ATC With Static Security Constraints

The static security constrained ATC problem is described as the remaining maximum amount of transferable power over the specified transmission line from an injection node (the generator bus) to an extraction node (the load bus) without violating any of static constraints. According to reference [7], static constraints will be bus voltage constraints, branch current constraints and generator reactive power constraints. The first incurred constraint among them sets the final value. Mathematically, ATC with static security can be stated as the following equations [6]:

$$f(x,\lambda)=\begin{bmatrix} f_P(x,\lambda) \\ f_Q(x,\lambda) \\ g(x) \end{bmatrix}=\begin{bmatrix} f_P(x)+\lambda d_P \\ f_Q(x)+\lambda d_Q \\ \max\left\{g_L(x),g_V(x),g_Q(x)\right\} \end{bmatrix}=0 \tag{11}$$

Where d_P, d_Q represent the direction vector of active and reactive power change; $f_P(x)$, $f_Q(x)$ are power mismatch; $g_L(x)$, $g_V(x)$ and $g_Q(x)$ represent the maximum limit value function of branch current constraints, bus voltage constraints and generator reactive power constraints; $\max\left\{g_L(x),g_V(x),g_Q(x)\right\}=0$ is the equivalent expression satisfying all constraints of static security, which means a certain constraint reaches its boundary limit; $g(x)$ is a pointwise maximum function, which has been proved that it is semismooth [8].

3.2 New ATC Model Including Wind Turbines

Let i be the node connected with wind turbines. By integrating (10) into (11), we construct the new ATC mathematical model with wind generation and saddle node bifurcation stability simultaneously as the following system of nonlinear equations:

$$F(x,s,\lambda)=\begin{bmatrix} F_P(x,\lambda) \\ F_Q(x,\lambda) \\ \Delta P_{\mathrm{m}} \\ g(x) \end{bmatrix}=\begin{bmatrix} F_p(x)+\lambda d_p \\ F_Q(x)+\lambda d_Q \\ P_{\mathrm{m}i}-P_{\mathrm{r}i}(V_i,s_i) \\ \max\left\{g_L(x),g_V(x),g_Q(x)\right\} \end{bmatrix}=0 \tag{12}$$

Where $F_P(x)$ and $F_Q(x)$ of nodes not connected with wind turbines are still same with $f_P(x)$ and $f_Q(x)$ of the static security constrained ATC. $F_P(x)$, $F_Q(x)$ of node i should be modified as $P_{ei}(V_i,s_i)-P_{Li}-V_i\sum_{j\in i}V_j(G_{ij}\cos\theta_{ij}+B_{ij}\sin\theta_{ij})$ and $Q_{ei}(V_i,s_i)-Q_{Li}-V_i\sum_{j\in i}V_j(G_{ij}\sin\theta_{ij}-B_{ij}\cos\theta_{ij})$.

4 A Smoothing Algorithm for Solving New ATC Model

The model (12) is a semi-smooth system of nonlinear equations since the maximum function (see reference [6] for semismooth concept). This results some conventional algorithms for nonlinear equations can not been applied. We will use smoothing strategy for the nonsmooth maximum function and set up a smoothing algorithm ([6], [8]).

4.1 The Smoothing Method for Maximum Function

The pointwise maximum function is defined in (13):

$$g(y)=\max_{1\le i\le m}\left\{g_i(y)\right\} \tag{13}$$

Where $g_i(y)$ is continuously differentiable function. For any $t > 0$, we define a smoothing function of $g(y)$ by

$$g_s(t,y) = \begin{cases} t\ln(\sum_{i=1}^{m} e^{g_i(y)/t}), & if\ t \neq 0 \\ \max_{1 \le i \le m}\{g_i(y)\}, & if\ t = 0 \end{cases} \tag{14}$$

Where t is called smoothing parameter. Clearly, $g_s(t,y)$ is smooth for $t > 0$. As t approaches zero, the smoothing function $g_s(t,y)$ approximates closer and closer to $g(y)$.

4.2 The Smoothing ATC Model

The smoothing method for maximum function is applied to smooth constraint functions of ATC model, constraint functions can be smoothed as:

$$g_s(t,x) = \begin{cases} t\ln(\sum_{i=1}^{N_{Branch}} e^{|i_i|-I_i^{\max}/t} + \sum_{j=1}^{N_{Load}} e^{V_j-V_j^{\max}/t} + \sum_{j=1}^{N_{Load}} e^{V_j^{\min}-V_j/t} + \sum_{k=1}^{N_{Gen}} e^{Q_k-Q_k^{\max}/t} + \sum_{k=1}^{N_{Gen}} e^{Q_k^{\min}-Q_k/t}), & t>0 \\ \max\{g_L(x), g_V(x), g_Q(x)\}, & t=0 \end{cases} \tag{15}$$

So, the semi-smooth ATC model (12) is smoothed as the following smoothing ATC mathematical model:

$$H(t,x,s,\lambda) = \begin{bmatrix} F_p(x)+\lambda d_p \\ F_Q(x)+\lambda d_Q \\ P_{mi}-P_{ri}(V_i,s_i) \\ g_s(t,x) \end{bmatrix} = 0 \tag{16}$$

This paper presents the new smoothing ATC model (16) considering static security and wind generation simultaneously. Obviously, when $t=0$, model(12) and (16) have the same solutions.

4.3 Calculation Method (Smoothing Newton Method)

Define $G(z) = G(t,x,s,\lambda) = \begin{bmatrix} t \\ H(t,x,s,\lambda) \end{bmatrix}$, then equations (12) and (16) have the same solutions with the following equations:

$$G(z) = 0 \tag{17}$$

Note that $G(z)$ is smooth as long as $t > 0$ and becomes nonsmooth (semi-smooth) when $t=0$. Suppose we solve (17) by an iterative method and the initial point $t = t^0 > 0$. From the first equation of (17), the next iteration $t^1 = t^0 + \Delta t = 0$, i.e., immediately goes to zero and H (and G) becomes nonsmooth. The iterative scheme can not proceed. To mitigate this problem, reference [9] has introduced a clever transformation of the function, which is adopted below.

Let $\bar{t}$ be a constant, $\bar{z} = [\bar{t}, 0]^{\mathrm{T}}$, and $\gamma \in (0,1)$ satisfying $\gamma\bar{t} < 1$, Define

$$\begin{cases}\psi(z)=\|G(t,x,s,\lambda)\|^2=t^2+\|H(t,x,s,\lambda)\|^2\\ \beta(z)=\gamma\min\{1,\psi(z)\}\end{cases} \tag{18}$$

The solutions to the following equations are all equivalent[9]:

$$G(z)=0\Leftrightarrow\beta(z)=0\Leftrightarrow G(z)=\beta(z)\bar{z} \tag{19}$$

In other words, solving (17) is equivalent to solving

$$G(z)=\beta(z)\bar{z} \tag{20}$$

Equation (20) will be solved by global smoothing Newton algorithm, as the following steps:

Step0.Choose the constant $\delta\in(0,1)$, $\sigma\in(0,0.5)$, $\bar{t}>0$ and $\gamma\in(0,1)$ such that $\gamma\bar{t}<1$.Let $z^0=(t^0,x^0,s^0,\alpha^0)$ be the initial point, $k=0$.

Step1. If $G(z^k)=0$, then stop. Otherwise let $\beta_k=\beta(z^k)$.

Step2. Solve equation $G(z^k)+G'(z^k)\Delta z^k=\beta_k\bar{z}$, solution $\Delta z^k=(\Delta t^k,\Delta x^k,\Delta s^k,\Delta\lambda^k)$ can be obtained.

Step3. Let l_k be the smallest nonnegative integer l satisfying

$$\psi(z^k+\delta^l\Delta z^k)\le\left(1-2\sigma(1-\gamma\bar{t})\delta^l\right)\psi(z^k). \tag{21}$$

Define $z^{k+1}=z^k+\delta^{l_k}\Delta z^k$.

Step4. Set $k=k+1$ and go to Step1.

5 Numerical Simulation Results and Analysis

In order to verify the feasibility of proposed model and algorithm, the IEEE 30-bus and 118-bus system connected to wind turbines are tested. The impact of wind turbines on ATC is analysed by comparing with simulation results of ATC model without wind turbines.

The data of wind turbines are as follows: rated terminal voltage $V_N=0.69\,\text{kV}$; $R_s+jX_s=0.00453+j0.0507\,\Omega$; $R_r+jX_r=0.00486+j0.1491\,\Omega$; $X_m=2.2059\,\Omega$; P_m can be obtained by solve equation (1). For convenience, this paper only calculation ATC problem in the rated mechanical output $P_m=600\text{KW}$, when rated wind speed $v_N=13.5\,\text{m/s}$.

5.1 Parameters in Algorithm

In the process of solving the smooth ATC model, set the parameters of smoothing Newton algorithm as follows: the initial point $z^0=(t^0,x^0,s^0,\lambda^0)$ with $t^0=\bar{t}$, x^0 is the solution of base-case power flow equations, $s^0=-0.004$ (the rated slip), $\lambda^0=10\text{MW}$; $\delta=0.5$, $\sigma=5\times10^{-5}$, $\gamma=0.2$, $\bar{t}=0.1$; the stopping criterion is

$G(z^k) \leq 1\times10^{-5}$.

5.2 Computation Results of IEEE system

Table 1: Partial calculation results of IEEE 30 bus system

Trade No.	Generator No.	Load No.	including wind turbines		not including wind turbines	
			iteration times	ATC(MW)	iteration times	ATC (MW)
1	2	3	86	55.2284	18	64.9766
2	2	4	86	44.6341	20	52.6036
3	2	6	84	30.1933	36	36.0357
4	5	10	47	30.4175	31	35.9177
5	5	12	46	42.3271	30	49.2476
6	8	4	64	140.2005	17	141.5585
7	8	16	83	62.2202	26	64.0100
8	8	17	89	92.6029	19	95.0813
9	11	19	94	46.5422	21	46.9146
10	11	21	82	61.6358	32	65.9401
11	13	4	88	98.8322	28	108.8092
12	13	25	86	27.6573	21	27.9764

Table 2: Partial calculation results of IEEE 118 bus system

Trade No.	Generator No.	Load No.	including wind turbines		not including wind turbines	
			iteration times	ATC(MW)	iteration times	ATC (MW)
1	25	5	93	56.9202	19	62.2211
2	25	28	90	50.6838	19	55.4742
3	46	21	126	31.9279	30	32.0459
4	46	37	94	89.2724	177	87.4156
5	59	16	93	79.5725	29	79.5920
6	59	28	183	70.6126	41	86.7544
7	76	83	136	8.8820	33	16.0558
8	80	60	92	93.3196	18	92.8535
9	80	117	97	46.9153	19	46.9169
10	91	98	91	56.2756	29	56.2443
11	107	88	91	30.3764	16	30.4203
12	113	21	125	32.2770	30	32.4019

28 wind turbines are connected to the node 4 via a step-up transformer in IEEE 30-bus system. The total trade cases of 30-bus system have 120 types for point-to-point ATC. In table 1, partial computation results of the ATC models are listed.

28 wind turbines are connected to the node 28 via a step-up transformer in IEEE 118-bus system. The total trade cases of 118-bus system have 3392 types for point-to-point ATC. In table 2, partial computation results of the ATC models are listed.

5.3 Analysis of Calculation Results

From Table 1 and 2, we can see that the iteration times of ATC model including wind turbines increase than the class ATC model not including wind turbines. This is because the new model increase one state variable and a balance equation, which needs to compute power flow equations and slip correction of wind turbines by turns, including the modification of Jacobian matrix.

Comparing the ATC values of Tables 1 and 2, it can be seen that, after wind turbines are connected to power system, there are changes in each point-to-point ATC:

1) ATC value is decreasing for the most points. But there exists some points to be increasing, such as the ATC values from 46 to 37, from 80 to 60, etc. in 118-bus system. In this regard, from the point of view of the mathematical model, although the ATC model with wind turbines increases an additional equality constraint, the power flow equations also have some changes.

2) Some point-to-point ATC have not obvious changes, such as the ATC values from 11 to 19, from 13 to 25, etc. in 30-bus system, the ATC values from 59 to 16, from 80 to 117, etc. in 118-bus system, this is because these load nodes are far away from the load node including wind turbines.

3) Some point-to-point ATC have more significant changes, such as the ATC values from 2 to 3, from 13 to 4, etc. in 30-bus system, the ATC values from 59 to 28, from 76 to 83, etc. in 118-bus system. The reason is that load nodes are near to, or connected with wind turbines. Therefore, the system operator should consider the remarkable changes of ATC at these nodes in order to deal with some special cases, such as the wind turbines to be cut off, or some faults happen, or the wind speed to be too low or too high, etc.

6 Conclusions

This paper sets up a new ATC model where the wind generation and static security are considered simultaneously. The new ATC model is a semismooth system of nonlinear equations. By using a smoothing strategy, a Newton-type solution algorithm is presented. Numerical results of the IEEE 30-bus and 118-bus system show the proposed model and algorithm are feasible and effective. Finally, the impact of wind turbines connected to power system on ATC is analysed by comparing with results of ATC model without wind turbines.

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