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# The Container Shipping Network Design under Changing Demand and Freight Rates

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**Abstract** This paper studies the optimization of container shipping network structures and its operations under changing cargo demand and freight rates. Most existing studies that use a given matrix of average demand have not been able to deal with practical issues such as empty container repositioning, ship-slot allocating, ship sizing, and container configuration. In this paper, these issues are simultaneously considered based on a series of matrices of demand in a year. The problem is formulated as a mixed integer non-linear programming problem (MINP) with an objective of maximizing the average unit ship-slot profit in three stages by using analytical methodology. A genetic algorithm based on heuristic is utilized to arrive at solutions. Through empirical experiments and comparisons, results show that the proposed model can provide a more realistic solution to the issues on the basis of changing demand and freight rates; the model provides a more effective approach to the optimization of container shipping network structures and operations than does a model based on average demand.

**Keywords** changing demand and changing freight rates; ship-slot allocating; empty container repositioning; container configuration; average unit ship-slot profit

## 1 Introduction

This paper addresses the issues of container shipping network structures and operations by taking into account changing demand along with changing freight rates in dealing with empty container repositioning, ship-slot allocating, ship sizing, and container configuration. With the growth of the global economy, the container shipping industry is playing a more important role in international cargo transportation. To adapt to greater container cargo shipment demand, shipping companies are increasing capacity via new super-size container ships. Companies also have begun to pay special attention to optimizing container shipping network designs and operations in order to promote higher quality service.

The container shipping network design problem (CSNDP) involves selecting a group of calling ports from a set of candidate ports and determining the calling sequence to be serviced by a fleet of ships with appropriate capacities. Calling ports are the sites that both deliver and pick up containers to and from one other. Cargo traffic demand along with freight rates at ports in a fixed service interval are usually known in advance. The objective, therefore, is to make optimal decisions regarding the following issues: voyage itinerary; the scale of ship assets and containers to be deployed; allocation of ship-slots at each calling port in a specified sequence;

container quantities loaded at each route; and maximizing ship-slot profits in a round-trip operation. The container shipping network operation should be a process in which ship and container movements at calling ports should proceed repeatedly during a planning period, and should be strictly characterized by a fixed interval, fixed ports, fixed ships, and a fixed route with published freight rates.

In most existing studies, CSNDP proposals are based on the assumption that cargo traffic demand is given only as a set of constants by a matrix of demand that represents either a set of stable values or a set of average values of annual demand in the ports. This assumption arises from the belief that ship sizes can be determined by the given demand, and that the costs of route shipping are fixed. It further assumes that freight rates are not directly affected by fluctuations in the real-world demand for a fixed ship capacity supply. But this assumption does not reflect the reality of container shipping network design. In fact, cargo traffic demand and freight rates do fluctuate periodically. These fluctuations are often significant, beyond the range of optimized network designs based on average demand. In this case, the shipping network operations may result in large losses when demand is at its lowest or reduced revenue when demand is higher. For example, in the Sino-Japan service, within a year the highest cargo traffic demand along with freight rates is often three times higher than the lowest demand and rates. Because of imbalanced directional cargo flows among calling ports, shipping companies transport a high number of empty containers. Thus the issues are as follows: how to move or lease empty containers in a timely and efficient manner; what size ships maximize revenues during peak seasons and minimize loss during off-seasons, and how to determine container configurations to reduce the risk of excessive containers in off-seasons and guarantee enough available containers in peak-seasons. These problems have already become critical and fundamental issues of the CSNDP, which should be influenced primarily by container cargo distribution among all ports in the trade area. Since it is essential to consider the impact of changing demand and changing freight rates, the CSNDP can be broken down into a series of sub problems including the ship routing problem (SRP), the calling sequencing problem (CSP), the ship-slot allocating problem (SAP), the ship-sizing problem (SSP), and the container constituting problem (RCCP).

Based on the characteristics and attributes of the CSNDP, this study will propose an integrated and average profit approach. This approach could optimize the whole problem by modeling the unit ship-slot profit of the network design in binary shipping directions with consideration of impacts of the fluctuation demands together with freight rates on the allocations of full and empty container flows among calling ports and the scales of ships and containers deployed in it. From the viewpoint of risk-revenue control, the proposed approach is superior to those based solely upon a matrix of average demands together with freight rates. Up until now, to the author's knowledge the approach of incorporating loss-revenue control with fluctuation demands has not been proposed in any existing CSNDP studies.

In fact, taking into account the interactions among the above factors and sub-problems, this paper develops a comprehensive method for the CSNDP; this method incorporates the cost of empty container repositioning, cost of ship sizing and container configuration, and revenues of ship-slot allocation and container quantity scheduling at calling ports into an optimal model with multi-stages. The problem is formulated using the analytical method and the average revenue expected value technique by the Knapsack problem (KP), Salesman Traveling problem (STP) and Mixed Integer Nonlinear problem (MIP) basis. The CSNDP is solved by a heuristic algorithm based on genetic algorithms (GA) with the objective of determining such decisive factors as a set of ports to be called, an order of calling sequence, a group of ship-slot allocations with container quantities handled at all calling ports, ship size, and container configurations deployed in service networks in order to maximize its average unit ship-slot profit on the condition of loss-risk control minimization. Finally, applying the problem of container transportation in a specific trade area, Far Asia, the numerical experiments and comparisons were conducted. Results show that the proposed formulation based on changing demand together with freight rates can provide a more realistic solution and an effective approach to the optimization problem of container shipping network structures and operations than those on the basis of average demands.

The rest of this paper is organized as follows: the relevant literature review is surveyed in Section Two. The problem description is presented in Section Three. The problem is formulated in Section Four. The computational experiments for the sample case are illustrated in Section Five. The study conclusion and recommendations for future research are discussed in Section Six.

## 2 Literature Review

A number of existing research papers have focused on container shipping transportation. Most can be sorted into two major categories: ship routing and related operations. These studies include the following review papers.

On the issue of containership routing, the existing literature is rather limited. A comprehensive survey of vehicle routing problems can be found in Bodin et al. (1983), Laporte (1992) and M. Christiansen et al. (2004). Boffey et al. (1979) developed a heuristic optimization model and an interactive decision support system for scheduling container ships on the North Atlantic route. Rana and Vickson (1988, 1991) tried to find the optimal sequence of calling ports for a fleet of ships operating on a trade route, in order to maximize the liner operation profit, while also determining an optimal calling port sequence. They assumed that non-profitable ports should be rejected as calling ports on the route. They formulated the problem as a mixed integer non-linear programming model and solved it by using Lagrangean relaxation techniques and decomposing method. Perakis and Jaramillo (1991) and Jaramillo and Perakis（1991) developed a Linear Programming model for a routing strategy to minimize total operating and lay-out cost over a planning time horizon. They also studied the assignment of an existing fleet of container ships to a predetermined set of routes (sequence of calling ports) based on realistic models of shipping operation costs. Cho and Perakis (1996) proposed the optimal models for fleet size and design of liner routes by taking into account future cargo demands both in real-life situations and future forecasts. The problem is formulated as a Mixed Integer Linear Programming and solved by devising a flow-route incidence matrix in the models to examine a number of candidate ports for the different ships. Fagerholt (1999) studied the problem of determining the optimal fleet and liner routes based on a weekly frequency, which was formulated as a multi-trip vehicle routing problem and was solved by a partitioning approach. Bendall and Stent (2001) proposed a determination model for the optimal fleet configuration while taking into account the fleet deployment plan applied in a hub-spoke container shipping network. Shao-wei Lam et al. (2007) used a simple shipping route with two ports and operation with two voyages (TPTV) and its extension with multiple ports and multiple voyages (MPMV) to demonstrate the effectiveness of an approximate dynamic programming approach in finding operational strategies for empty container allocation. Since the temporal difference learning for average cost minimization is utilized in the proposed approach, only two voyages may not be sufficient to represent a complete shipping route system operation. Chaug-Ing Hsu and Yu-Ping Hsieh (2004) formulated a two-objective model to determine the optimal liner routing, ship size, and sailing frequency for carriers and shippers by minimizing shipping costs and inventory costs simultaneously, based on a trade-off between the two costs. From the viewpoints of carriers and shippers, the proposed approach may be of practical value.

On the issue of shipping route operations, considerable research has been done, focused primarily on empty container repositioning. Gavish (1981) developed a system for making decisions regarding container fleet management. In his study, if the empty containers were not relocated at the requested time, the system would assign the owned and leased containers to satisfy the demand based on the marginal cost criterion. It should be further noted that the extra leased containers affected the liner operation total cost without consideration of the inventory of idle owned containers. Crainic et al. (1993) proposed dynamic and stochastic models for empty container relocation in a land distribution and transportation system. Similarly, to deal with the problem of leased container allocation and empty container relocation, the authors ignored the difference between short-term leasing cost and long-term cost. This seems impractical and not in keeping with the practice of dealing with long-term leased containers as owned ones. Cheung and Chen (1998) also considered the sea-borne empty container allocation problem. In their paper, the dynamic container allocation problem was formulated as a two-stage stochastic network model. The model assists liner operators in allocating empty containers and consequently in reducing leasing cost and inventory level at calling ports. However, their work failed to consider the duration of leasing time. Koichi Shintani et al. (2009) studied the optimization problem for container shipping network design, proposing an approach to solve the empty container repositioning problems. In their paper, the port calling sequence and empty container repositioning are considered simultaneously by designing the objective function with a penalty cost factor. Thus, the problem is integrated and formulated as a two-stage problem. The idea of adding penalty cost in the proposed model and using virtual points in designing networks structure should be certainly valuable. But, due to a lack of cargo traffic demand fluctuations and cargo flow distributions among ports in their experiments, there are evident flaws in the ship-slot allocations on board the ship at calling ports. More recently, Hwan Chang et al. (2008) studied a heuristic method to provide an optimal solution to reduce the cost of empty container interchange. Using available data, they tested the effectiveness of computational time and solution quality. Massimo Di Francesco et al. (2009) developed a multi-scenario, multi-commodity, time-extended optimization model to deal with empty container repositioning problem. Some uncertain parameters in the model, which cannot be estimated through historical data, are treated as sets of a limited number of values, according to shipping company opinions. Bandeira, D.L., et al. (2009) proposed a decision support system (DSS) to deal with full and empty container trans-shipment operations. The arrangement of repositioning empty containers can be determined by adjusting several parameters in the DSS model.

None of the above studies has addressed the proposed problem, as well as the approach in this paper: namely that container shipping network design and operation should be incorporated into a single, coordinated problem to be addressed by considering the revenue-loss risk control of ship sizing and container configuration based on periodic fluctuations of cargo traffic demand together with freight rates.

## 3 Problem Description

Essentially, the CSNDP corresponds to the problem of maximizing profits while shipping container cargoes at calling ports. In general, the optimization should be completed by a series of decision-making processes that involve selecting appropriate calling ports from candidate ports in a trade area, determining the reasonable order of calling sequence with a fixed regular frequency service, and settling the rational ship-slot allocation at each calling port with the suitable scale of deployed assets that include ship size, container quantity, and container configurations in the network. These decision-making processes depend upon the following influencing factors, also called controllable factors, which mainly cover the distances, cargo traffic demand together with freight rates among candidate ports in a trade area, investment costs of ships and containers, and a company's policies regarding shipping market and investment, etc. Based on these controllable factors, the decision-making process ought to determine factors including the optimal set of ports to be called, the optimal order of calling sequence, the optimal size of ships, and the optimal series of ship-slot allocations on shipboard at each calling ports. Since the ship size is unchangeable during a planning period and fluctuating demand produces a significant effect on ship size, it is more feasible to use a series of matrices of demand in order of time to represent fluctuating demand rather than to use only a matrix of average demand.

The fundamental form of container shipping network structures and operations can be described as follows. The container shipping network structures generally can be divided into two types of forms according to their operation characteristic. One is called the circular and another is called the pendulum, shown in Figure.1. From the viewpoint of topology, they can be essentially reduced to the circular route as a basic form, because any pendulum type can be converted to a circular one by adding virtual points which represent the ports in the backward direction and by constructing an adequate matrix for demand distributions. The shipping network operation is generally performed by a fleet of ships with a series of ship-slot allocations for calling ports. The fleet of ships traveling on the route ought to be split into two groups -- one group travels in a clockwise direction while another travels at the same time in a counterclockwise direction. In this way, the cargo traffic at any calling port is conveniently transported to its adjacent ports in different directions. For example, the cargo traffic from Port 1 to Port 2 must be carried by one group of ships in the clockwise direction, and the cargo traffic from Port 1 to Port 9 can be carried by another group of ships in the counterclockwise direction, as shown in Figure.2. The ships are only required to pick up the containers to be transported to other calling ports that are located in a half voyage traveling in the same direction as the ships.

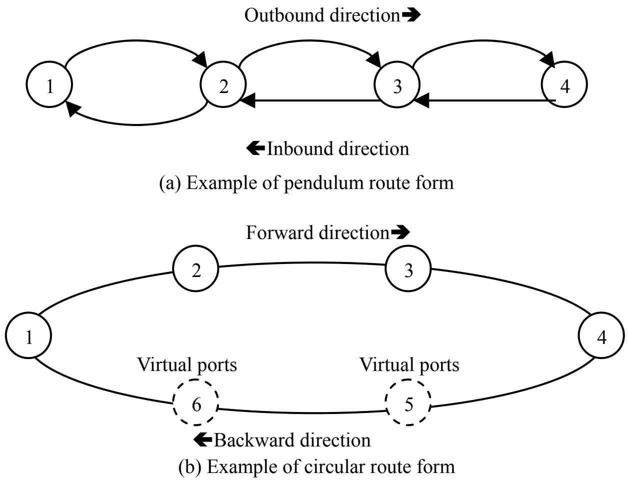


Figure 1: Conversion of pendulum route form to circular

In addition, due to the imbalanced directional cargo flows between some calling ports, there must exist a difference between the total cargo traffic originating from the port and the one arriving to it on the adjacent voyages and the demands for empty containers at any port in the requested time. Since load rejection is very unlikely in practice assuming that ships' capacities have spare slots, liner shipping companies must decide whether to reposition empty containers or lease extra containers and store idle owned containers at the specific ports. Since comparisons of the costs in a single voyage are not reasonable, comparison of these average costs in a sufficient number of voyages under consideration are necessary. These elements must be represented in the formulation as the opportunity costs with the mutual-substitution relation between them.

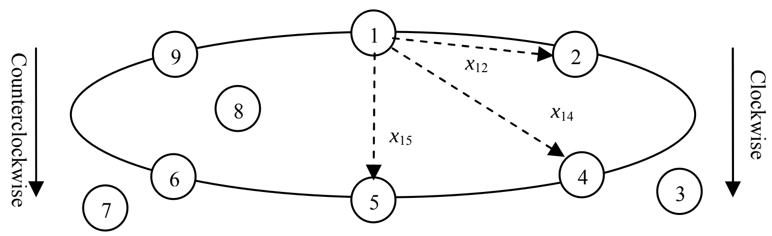


Figure 2: Example of circular route operation

Thus, the model we will construct should include the above influencing factors and elements. The model with an objective of the average unit ship-slot profit maximization can be formulated by designing an average closed voyage trip in a circular route with appropriate scales of ships and containers deployed. In the ship routing, it is not necessary for ships to call at all ports in the trade area, for example, ports 1, 2, 4, 5, 7 and 9 are selected except ports 3, 7, and 8, as shown in Figure 2. Other assumptions are as follows:

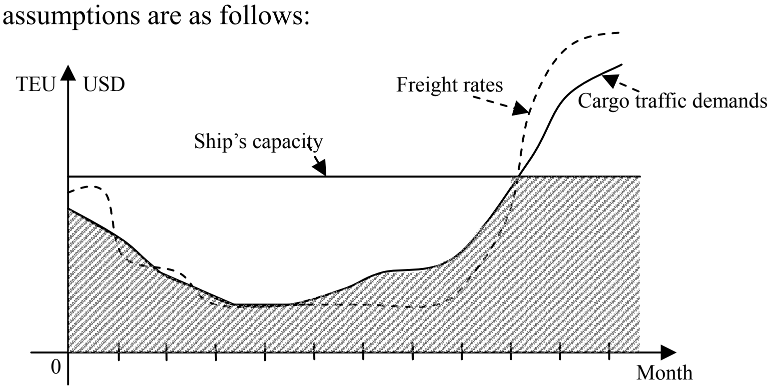


Figure 3: periodic fluctuation of cargo traffic demand and freight rates

(a) As a key influencing factor, fluctuating cargo traffic demand among all ports ought to be presented by a series of demand matrices in order of time, with relevant homogenous freight rates, rather than by a matrix of average demands in a planning horizon. The reason is mainly because the practical number of containers transported in the planning horizon should be limited by the accepted ship size once it is determined, as shown in Figure 3.

(b) There must be an appropriate quantity of containers equipped at every calling port corresponding to the quantity handled at them, according to container shipping network design. Additional containers can be leased at any port, but they must ultimately be returned to the original port.

(c) The ships deployed in the network or route must be the same with capacities and cruising speeds.

(d) The ship's capacity must not be exceeded by total number of containers loaded on shipboard at any route leg.

## 4 MINP Formulation

As we described, the CSNDP consists of four sub-problems. The first is to choose the best group of calling ports for the optimal network or route. The second is to identify the calling sequence of the chosen group of calling ports for an optimal arrangement of voyage itinerary. The third is to optimize ship-slot allocations at each calling ports with a series of container quantities handled on each voyage at each calling port for the average unit ship-slot profit maximization. The last is to determine rational container configurations deployed in networks depending upon the above container quantities handled at each calling port. Since there exists the interrelations and interactions among these sub-problems, the CSNDP can be formulated as a mixed integer non-linear programming problem (MINP) in three stages, based respectively on a Knapsack Problem (KP), a Salesman Traveling Problem (STP) with an Operation Problem (OP) and a Container Configuration Problem (CCP). The optimal model can be developed as follows:

**Stage 1:**

[**KP**] Maximize $\sum_{k\in V} \omega_k \times P^k$, (1)

Subject to $\sum_{k\in V} \omega_k = 1$, (2)

$\omega_k \in \{0,1\}, \qquad \forall k \in V,$ (3)

Where

$V$ Set of combinations of calling ports, which are taken from a set of candidate ports *N* in trade area;

$\omega_k$ =1 if the route constructed by a candidate combination of calling ports *k* is selected, =0 otherwise;

$P^k$ Values of objective function under the candidate combination of calling ports *k*.

**Stage 2:**

Given a set of calling ports, an optimal calling sequence can be formulated by constructing the MINP with the STP and the OP. In order to find the decision variables as we described, let $w_{ij}$ $(i, j \in N, i \neq j)$ binary flow variables, $x_{ij}, y_{ij}$ $(i, j \in N, i \neq j)$ be respectively full and empty ship-slot allocation variables at each calling port, *u* be the ship-size variable and $X_{ijg}, Y_{ijg}$ $(i, j \in N, g \in G)$ express respectively the real quantities of full and empty containers, as auxiliary variables, loaded in the scenario $g$ $(g \in G)$ of series of cargo traffic demand $d_{ijg}$ $(i, j \in N, g \in G)$. In consideration of the period fluctuations of cargo traffic among calling ports, the unit ship-slot profit an average voyage in a planning horizon is introduced, which may be more reasonable and effective and can be represented by the expected revenue in a year with total *G* voyages. Thus, if the route operation by only one ship with the capacity ($u$) is considered, [MINP] may be formulated by the unit ship-slot profit an average voyage in a year with total *G* voyages as follows:

[**MIP**] Maximize

$$\overline{P} = \frac{1}{G \cdot u}\left( \sum_{i\in N}\sum_{j\in N}\sum_{g\in G} [(R_{ij}^f - C_{ij}^f)\cdot X_{ijg} + w_{ij}\cdot(R_{ij}^e - C_{ij}^e)\cdot Y_{ij}] \right.$$

$$\left. - \sum_{i\in N}\sum_{j\in N} t_{ij}\cdot w_{ij}\cdot\left(\sigma\cdot(\frac{G+1}{2})\cdot[\lambda\cdot LS_i + \mu\cdot ST_i] + C_{cn} + C_{sh}(u)\right)\right), \quad (4)$$

Subject to

$$\sum_{j\in N} w_{ij} - \sum_{j\in N} w_{ji} = 0, \qquad \forall i \in N, \tag{5}$$

$$\sum_{i\in\phi}\sum_{j\notin\phi} w_{ij} \geq 1, \qquad \forall \phi \subset N, \tag{6}$$

$$w_{ij} \in \{0,1\}, \qquad \forall i, j \in N, \tag{7}$$

$$z_{ij} \leq d_{ijg}, \qquad \forall i, j \in N, \ \forall g \in G, \tag{8}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} (x_{ij} + y_{ij}) a_{ijm} \leq u, \qquad \forall i, j \in N, \forall m \in M, \forall g \in G, \tag{9}$$

$$X_{ijg} = d_{ijg} \qquad \forall x_{ij} \geq d_{ijg}, \ \ i, j \in N, g \in G, \tag{10}$$

$$X_{ijg} = x_{ij}, \ \ Y_{ijg} = y_{ij} \qquad \forall x_{ij} < d_{ijg}, \ \ i, j \in N, g \in G, \tag{11}$$

$$x_{ij}, y_{ij}, X_{ijg}, Y_{ijg} \in N \cup \{0\}, \ \ \forall i, j \in N, \ \ \forall g \in G, \tag{12}$$

$$L_i = \max\{[\sum_{j\in N}(X_{ji} + Y_{ji}) - \sum_{j\in N}(X_{ij} + Y_{ij})], 0\} \ \ \forall i, j \in N, \tag{13}$$

$$S_i = \max\{[\sum_{j\in N}(X_{ij} + Y_{ij}) - \sum_{j\in N}(X_{ji} + Y_{ji})], 0\} \ \ \forall i, j \in N, \tag{14}$$

where

$$C_{cn} = \eta \cdot \sum_{i\in N}\{\max\{\sum_{j\in N}(x_{ij} + y_{ij}), \sum_{j\in N}(x_{ji} + y_{ji})\}\} + v \times u, \tag{15}$$

$$C_{sh}(u) = (\alpha \cdot u^2 + \beta \cdot u + \gamma) \times (v + \varepsilon \cdot n), \tag{16}$$

$t_{ij}$ Time of ship's travel between port ( $i$ ) to port ( $j$ );

$v$ Number of ships deployed in the route with a fixed cruising speed;

$N$ Set of calling ports for $k \in V$ ;

$\phi$ Non-empty subset of $N$;

$C_{sp}(\cdot)$ Shipping cost function of selected arcs ( $i$, $j$ ), which is represented in a linear part of the scope;

$C_{cn}(\cdot)$ Requested quantity of containers deployed in the route (including at ports and on the shipboard);

$LS_i$ Number of leasing containers (TEU) at port $(i)$;

$ST_i$ Number of storing containers (TEU) at port $(i)$;

$M$ Number of route legs which equal to the number of calling ports in the circular route form;

$C_{ij}^f, C_{ij}^e$ Unit cost of handling full and empty containers (TEU) at calling port;

$R_{ij}^f, R_{ij}^e$ Unit revenue of transporting full and empty containers (TEU) from port $(i)$ to port $(j)$;

$a_{ijm}$ =1 if the route-leg covering the cargo traffic flows between port pairs $(i, j)$, =0 otherwise.

The functions (1)-(3) provide the method by which the optimal set of ports to be called in the route can be selected from the candidate ports in the trade area. Constraint (5) ensures that each ship that arrives at a calling port must leave from it. Constraint (6) gives a guarantee that all the ports to be called must be connected via the constructed route in which there is no such sub-voyage that it does not visit all the ports selected in *N*. constraints (8)-(9) are constraints for full containers loaded at each calling port and ship's capacity on any route-leg, respectively. Constraints (10)-(12) indicate the real quantity of containers loaded at each calling port equals to the real cargo traffic demand when the quantity of ship-slots allocated on shipboards is greater than real cargo traffic demand at it, and otherwise, the real quantity of containers loaded at the port only equals to the quantity of ship-slots allocated on shipboards. Constraints (13) and (14) are leasing and storing container constraints. Functions (15)-(16) represent the relationships between the assets of ships and containers deployed in the route and shipping operation cost with the ship size, where $\eta, \alpha, \beta, \gamma$ respectively denote the weighted factors for the relative terms. The objective function (4) is to maximize the unit ship-slot profit of an average voyage, which is an algebraic sum of the total revenue, repositioning cost, leasing and storage costs, and assets operation costs divided by ship's capacity, where $\sigma, \lambda, \mu, \varepsilon$ express the cost coefficients of the relative terms, respectively.

**Stage 3:**

The container configuration problem is to determine and arrange the optimal configurations of containers with owned container quantity, long-term leasing container quantity, and short-term leasing container quantity deployed in networks in order to minimize the total using container cost. If the short-term leasing time be set to less than three months and the long-term leasing time be more than three months, and let $Q_i^O$, $Q_i^L$ and $Q_i^S$ respectively signify quantity variables of owned containers, long-term leasing containers and short-term leasing containers deployed at calling port $i$, the total container cost involving using costs and idling costs of containers can be formulated as follows:

[**CCP**] Minimize

$$C = \sum_{i \in N} (C^O \times Q_i^O + C^L \times Q_i^L + C^S \times Q_i^S) + \sum_{i \in N} (\Delta^O \cdot C_{I\ i}^O + \Delta^L \cdot C_{I\ i}^L + \Delta^S \cdot C_{I\ i}^L), \tag{17}$$

Subject to:

$$Q_i^O + Q_i^S + Q_i^L \le \sum_{\substack{j \in N \\ j \ne i}} (x_{ij} + y_{ij}), \qquad \forall i, j \in N, \tag{18}$$

$$C^O \le C^L \le C^S, \quad (19)$$

$$0 \le Q_i^O \le \min_{g\in G}\{\sum_{\substack{j\in N\\ j\ne i}}(X_{ijg}+Y_{ijg})\}, \qquad \forall i,j\in N, \forall g\in G, \quad (20)$$

$$0 \le Q_i^L \le (\frac{1}{G}\sum_{\substack{j\in N\\ j\ne i}}\sum_{g\in G}(X_{ijg}+Y_{ijg}) - Q_i^O), \qquad \forall i,j\in N, \forall g\in G, \quad (21)$$

$$0 \le Q_i^S \le (\max_{g\in G}\{\sum_{\substack{j\in N\\ j\ne i}}(X_{ijg}+Y_{ijg})\} - \frac{1}{G}\sum_{\substack{j\in N\\ j\ne i}}\sum_{g\in G}(X_{ijg}+Y_{ijg})), \quad \forall i,j\in N, \forall g\in G, \quad (22)$$

$$\Delta^O = Q_{ig}^O - \sum_{\substack{j\in N\\ j\ne i}}(X_{ijg}+Y_{ijg}), \qquad \forall i,j\in N, \forall g\in G, \Delta^O \ge 0, \quad (23)$$

$$\Delta^L = (Q_{ig}^O + Q_{ig}^L) - \sum_{\substack{j\in N\\ j\ne i}}(X_{ijg}+Y_{ijg}), \qquad \forall i,j\in N, \forall g\in G, \Delta^L \ge 0, \quad (24)$$

$$\Delta^L = (Q_{ig}^O + Q_{ig}^L + Q_{ig}^S) - \sum_{\substack{j\in N\\ j\ne i}}(X_{ijg}+Y_{ijg}), \quad \forall i,j\in N, \forall g\in G, \Delta^S \ge 0, \quad (25)$$

The function (17) represents the total using cost of all containers during an average voyage. Where are coefficients $C^O$, $C^L$ and $C^S$ which respectively imply unit using costs of owned containers, long-term leasing containers, and short-term leasing containers. And where are variables $\Delta^O$, $\Delta^L$ and $\Delta^S$ which signify the idle quantities of owned containers, long-term leasing containers, and short-term leasing containers deployed in the route, respectively. Homogenously, there are coefficients $C_{I\ i}^O$, $C_{I\ i}^L$ and $C_{I\ i}^S$ which respectively denote unit idle costs of owned containers, long-term leasing containers, and short-term leasing containers. Constraint (18) gives the limit requirement between quantities handled and quantities of all container configurations deployed at each calling port. Inequality (19) shows the requirement of general relations among $C^O$, $C^L$ and $C^S$. Constraints (20)-(22) represent constraint requirements between variables deployed and quantities handled at each calling port. Equalities (23)-(25) represent the idle quantity of each part of container configurations at every calling port, respectively.

# 5 Case Experiments

This section presents sample cases to demonstrate the application of the proposed formulation. The case experiments focus on the container shipping network design in the trade area of Far East Asia. Since there are a number of relevant factors to be considered that impact the shipping network design, we implemented the case experiments according to some empirical knowledge about shipping operation and management in this trade area.

The case experiments are implemented in a heuristic based on genetic algorithm (GA) which is programmed by the Mat Lab software technique. First, depending on the periodic fluctuations of historic cargo traffic demand, the relatively uncontrollable factors, which are feasible variables such as an optimal set of calling ports, an optimal order of calling sequence, ship size, as well as ship-slot allocation, quantities handled at calling ports, are found by the proposed formulations (1)-(4) with constraints (5)-(16). Then, based on the found feasible variables, the optimal container configurations with owned container quantity, long-term container quantity and short-term container quantity at each calling port respectively are obtained by the proposed functions (17) with constraints (18)-(25). Meanwhile, in order to assess the solution effects of the formulations under fluctuating demand, we compare the approximate solutions of the formulations of the SCNDP based on fluctuating demand by the GA with the ones of the formulations of the SCNDP based on average demand by the GA. Taking into consideration computational limitations of these methods, we tested the problem using the cases of ten candidate ports in the trade area. Results show that the optimal solutions to the proposed formulation of the CSNDP based on fluctuating demand are better than the ones based on average demand to the objective reality.

## 5.1 Parameter Settings

(1) Candidate ports in the trade area (10 ports): Dalian, Tianjin, Qingdao, Shanghai, Busan, Kaohsiung, KeeLung, Kitakyushu, Osaka, and Tokyo.

(2) The planning horizon: one year.

(3) The weekly service frequency: once.

(4) The turnaround time of containers at each port: less than or equal to service interval.

(5) The storage cost at each port ($i$): $USD2/TEU • day.

(6) The short-term leasing cost at each port ($i$): $USD2/TEU • day.

(7) Given ship cruising speed: 21 knots.

(8) Total handling and standby time at each port: 0.5 day/per port.

(9) The given cargo traffic demand in matrix: from January to December.

(10)Fuel oil and diesel oil cost: $USD 320 /metric ton and $USD 560 /metric ton respectively.

The above parameters (5)-(8) are set to be average value. Due to the lack of detailed data about ship expense criteria at each port, we assume that they are the same for all the ports under consideration. However, according to such assumptions, the reliability of the solution cannot be affected in the decision making.

## 5.2 Demand Matrices

With the characteristic of periodic fluctuations, historic cargo traffic demand may be obtained through market surveys or provided by liner companies. The distributions of cargo traffic demand with relevant freight rates can be represented by a series of matrices which consist of weekly data based on bi-months in a year. The series of matrices for fluctuating demands of this case are given in the following tables.

Table 1: Weekly distributions of average demand and freight rates in January and February (TEU/USD)

| $d_{ij6} R_{ij6}^f$ | DL | TJ | QD | SH | BSN | OSK | KTK | TKY | KL | KHS |
|---|---|---|---|---|---|---|---|---|---|---|
| DL | 0/0 | 0/0 | 0/0 | 0/0 | 230/250 | 220/250 | 200/250 | 220/230 | 250/430 | 200/440 |
| TJ | 0/0 | 0/0 | 0/0 | 0/0 | 240/270 | 230/260 | 220/260 | 250/240 | 270/450 | 250/430 |
| QD | 0/0 | 0/0 | 0/0 | 0/0 | 250/240 | 230/240 | 240/230 | 220/240 | 260/420 | 250/420 |
| SH | 0/0 | 0/0 | 0/0 | 0/0 | 280/260 | 220/250 | 280/230 | 250/250 | 260/380 | 250/400 |
| BSN | 105/230 | 125/220 | 200/220 | 200/220 | 0/0 | 250/270 | 300/250 | 300/270 | 350/330 | 300/350 |
| OSK | 50/230 | 40/250 | 40/240 | 50/240 | 60/260 | 0/0 | 0/0 | 0/0 | 170/350 | 190/380 |
| KTK | 20/220 | 30/240 | 40/220 | 40/220 | 150/250 | 0/0 | 0/0 | 0/0 | 160/300 | 150/300 |
| TKY | 25/230 | 35/250 | 30/230 | 55/240 | 40/260 | 0/0 | 0/0 | 0/0 | 145/350 | 125/380 |
| KL | 233/370 | 350/380 | 320/360 | 420/340 | 350/450 | 325/380 | 300/360 | 330/380 | 0/0 | 0/0 |
| KHS | 260/390 | 240/380 | 300/380 | 400/340 | 340/460 | 300/360 | 300/360 | 290/360 | 0/0 | 0/0 |

Table 2: Weekly distributions of average demand and freight rates in March and April (TEU/USD)

| $d_{ij6} R_{ij6}^f$ | DL | TJ | QD | SH | BSN | OSK | KTK | TKY | KL | KHS |
|---|---|---|---|---|---|---|---|---|---|---|
| DL | 0/0 | 0/0 | 0/0 | 0/0 | 100/220 | 200/230 | 180/220 | 200/210 | 200/300 | 200/300 |
| TJ | 0/0 | 0/0 | 0/0 | 0/0 | 160/220 | 200/230 | 200/220 | 220/240 | 220/320 | 220/330 |
| QD | 0/0 | 0/0 | 0/0 | 0/0 | 160/210 | 150/220 | 160/210 | 160/220 | 220/360 | 250/380 |
| SH | 0/0 | 0/0 | 0/0 | 0/0 | 210/160 | 200/160 | 260/160 | 250/160 | 260/340 | 240/350 |
| BSN | 55/220 | 75/230 | 100/210 | 160/210 | 0/0 | 230/240 | 210/240 | 220/240 | 250/380 | 250/400 |
| OSK | 30/220 | 30/240 | 30/220 | 40/200 | 50/280 | 0/0 | 0/0 | 0/0 | 100/320 | 120/320 |
| KTK | 20/220 | 30/240 | 30/220 | 28/200 | 45/280 | 0/0 | 0/0 | 0/0 | 110/350 | 100/350 |
| TKY | 25/220 | 30/240 | 30/220 | 45/200 | 40/280 | 0/0 | 0/0 | 0/0 | 115/350 | 100/350 |
| KL | 260/280 | 250/280 | 240/270 | 360/240 | 300/400 | 220/30 | 220/280 | 220/300 | 0/0 | 0/0 |
| KHS | 220/280 | 210/280 | 200/270 | 300/240 | 300/400 | 240/300 | 220/280 | 240/300 | 0/0 | 0/0 |

Table 3: Weekly distributions of average demand and freight rates in May and June (TEU/USD)

| $d_{ij6} R_{ij6}^f$ | DL | TJ | QD | SH | BSN | OSK | KTK | TKY | KL | KHS |
|---|---|---|---|---|---|---|---|---|---|---|
| DL | 0/0 | 0/0 | 0/0 | 0/0 | 200/200 | 200/210 | 240/200 | 240/220 | 220/280 | 240/280 |
| TJ | 0/0 | 0/0 | 0/0 | 0/0 | 230/210 | 240/210 | 210/210 | 220/220 | 230/300 | 230/310 |
| QD | 0/0 | 0/0 | 0/0 | 0/0 | 180/180 | 250/200 | 250/180 | 280/200 | 240/360 | 280/360 |
| SH | 0/0 | 0/0 | 0/0 | 0/0 | 250/180 | 280/200 | 300/180 | 280/200 | 280/340 | 260/340 |
| BSN | 50/200 | 75/210 | 150/210 | 220/210 | 0/0 | 220/210 | 200/180 | 200/210 | 200/350 | 200/360 |
| OSK | 30/200 | 30/210 | 30/200 | 50/180 | 50/200 | 0/0 | 0/0 | 0/0 | 100/300 | 100/300 |
| KTK | 20/200 | 30/220 | 35/200 | 50/180 | 45/190 | 0/0 | 0/0 | 0/0 | 110/320 | 100/320 |
| TKY | 25/200 | 30/210 | 30/200 | 45/180 | 40/200 | 0/0 | 0/0 | 0/0 | 100/320 | 90/320 |
| KL | 200/280 | 200/280 | 250/270 | 380/240 | 340/380 | 255/300 | 250/300 | 240/300 | 0/0 | 0/0 |
| KHS | 200/280 | 160/280 | 210/270 | 340/240 | 320/380 | 200/300 | 230/320 | 240/300 | 0/0 | 0/0 |

Table 4: Weekly distributions of average demand and freight rates in July and August (TEU/USD)

| $d_{ij6} R_{ij6}^f$ | DL | TJ | QD | SH | BSN | OSK | KTK | TKY | KL | KHS |
|---|---|---|---|---|---|---|---|---|---|---|
| DL | 0/0 | 0/0 | 0/0 | 0/0 | 230/200 | 230/370 | 390/300 | 360/320 | 280/500 | 270/480 |
| TJ | 0/0 | 0/0 | 0/0 | 0/0 | 200/220 | 260/390 | 300/310 | 360/320 | 270/480 | 300/490 |
| QD | 0/0 | 0/0 | 0/0 | 0/0 | 210/210 | 300/320 | 350/300 | 360/320 | 300/450 | 350/470 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **SH** | 0/0 | 0/0 | 0/0 | 0/0 | 360/220 | 390/320 | 400/300 | 400/320 | 350/430 | 340/450 |
| **BSN** | 105/250 | 125/250 | 200/250 | 220/300 | 0/0 | 280/380 | 330/280 | 350/380 | 350/400 | 300/400 |
| **OSK** | 50/230 | 40/240 | 40/210 | 50/220 | 60/280 | 0/0 | 0/0 | 0/0 | 210/350 | 190/380 |
| **KTK** | 20/210 | 30/220 | 40/200 | 40/200 | 100/280 | 0/0 | 0/0 | 0/0 | 200/300 | 180/300 |
| **TKY** | 25/220 | 35/240 | 30/210 | 55/220 | 80/260 | 0/0 | 0/0 | 0/0 | 190/350 | 185/380 |
| **KL** | 280/320 | 250/330 | 300/300 | 410/280 | 400/520 | 325/490 | 350/480 | 330/490 | 0/0 | 0/0 |
| **KHS** | 260/310 | 240/320 | 300/300 | 400/280 | 380/500 | 320/490 | 320/480 | 300/490 | 0/0 | 0/0 |

Table 5: Weekly distributions of average demand and freight rates in September and October (TEU/USD)

| $d_{ij6}R_{ij6}^f$ | DL | TJ | QD | SH | BSN | OSK | KTK | TKY | KL | KHS |
|---|---|---|---|---|---|---|---|---|---|---|
| **DL** | 0/0 | 0/0 | 0/0 | 0/0 | 280/300 | 280/500 | 400/490 | 400/540 | 360/520 | 320/540 |
| **TJ** | 0/0 | 0/0 | 0/0 | 0/0 | 240/320 | 260/500 | 440/500 | 450/550 | 390/540 | 400/540 |
| **QD** | 0/0 | 0/0 | 0/0 | 0/0 | 280/320 | 300/500 | 500/460 | 480/550 | 400/500 | 400/500 |
| **SH** | 0/0 | 0/0 | 0/0 | 0/0 | 400/350 | 560/480 | 550/460 | 560/540 | 450/460 | 400/480 |
| **BSN** | 120/370 | 150/380 | 210/380 | 300/400 | 0/0 | 550/540 | 500/350 | 480/560 | 450/460 | 400/480 |
| **OSK** | 60/240 | 50/250 | 50/240 | 60/240 | 80/310 | 0/0 | 0/0 | 0/0 | 200/400 | 210/400 |
| **KTK** | 30/220 | 40/240 | 50/220 | 50/220 | 100/300 | 0/0 | 0/0 | 0/0 | 200/360 | 200/370 |
| **TKY** | 35/240 | 45/250 | 40/240 | 65/240 | 50/310 | 0/0 | 0/0 | 0/0 | 165/400 | 145/400 |
| **KL** | 250/430 | 300/450 | 350/420 | 510/420 | 500/680 | 425/600 | 450/580 | 430/600 | 0/0 | 0/0 |
| **KHS** | 280/430 | 300/450 | 320/420 | 500/420 | 500/680 | 420/600 | 400/580 | 410/610 | 0/0 | 0/0 |

Table 6: Weekly distributions of average demand and freight rates in November and December (TEU/USD)

| $d_{ij6}R_{ij6}^f$ | DL | TJ | QD | SH | BSN | OSK | KTK | TKY | KL | KHS |
|---|---|---|---|---|---|---|---|---|---|---|
| **DL** | 0/0 | 0/0 | 0/0 | 0/0 | 360/330 | 530/600 | 560/600 | 560/650 | 450/620 | 370/610 |
| **TJ** | 0/0 | 0/0 | 0/0 | 0/0 | 400/350 | 560/650 | 580/650 | 560/650 | 500/650 | 500/620 |
| **QD** | 0/0 | 0/0 | 0/0 | 0/0 | 380/380 | 600/620 | 600/600 | 620/620 | 500/600 | 460/600 |
| **SH** | 0/0 | 0/0 | 0/0 | 0/0 | 450/400 | 660/620 | 670/600 | 650/620 | 540/550 | 500/550 |
| **BSN** | 110/400 | 140/450 | 300/400 | 300/450 | 0/0 | 480/470 | 600/450 | 500/470 | 500/500 | 420/550 |
| **OSK** | 60/240 | 50/270 | 50/260 | 50/250 | 120/340 | 0/0 | 0/0 | 0/0 | 190/420 | 210/440 |
| **KTK** | 40/220 | 40/250 | 70/240 | 60/240 | 150/320 | 0/0 | 0/0 | 0/0 | 210/400 | 250/420 |
| **TKY** | 40/240 | 45/270 | 40/260 | 65/250 | 120/340 | 0/0 | 0/0 | 0/0 | 190/420 | 225/440 |
| **KL** | 480/470 | 450/480 | 440/500 | 600/500 | 600/700 | 525/680 | 600/660 | 530/680 | 0/0 | 0/0 |
| **KHS** | 360/470 | 440/480 | 400/500 | 550/500 | 580/700 | 520/620 | 550/660 | 540/650 | 0/0 | 0/0 |

## 5.3 Experiments' Results

Ship-size, as a main variable, should be represented by the relevant function with shipping cost. Actually, it is very difficult to construct the precise relationship between ship-size and shipping cost with an exact function. Generally, the function should be presented by a quadratic approximation. Taking into consideration cargo traffic in the trade area of the case, the ship-size to be deployed may be located in the categories of 2,000 TEU to 7,000TEU. In this section, the relationship between ship-size and shipping cost can be represented approximately by a linear function. The function consists of three sub models: one is associated with fuel oil and diesel oil consumption, the second is associated with ship leasing, and the last is associated with cost of handling the ship at calling ports. By quoting relevant models and performing the regression analysis based on the above cost data, we set up the following linear shipping cost function using the TEU (twenty-foot equivalent unit) capacity as the independent variable:

$$C_{sp} = 9.54 \cdot u + 21{,}973.38$$

Table 7: Optimal ship-slot allocations at each calling port based on fluctuating demand

| $X_{ij},Y_{ij}$ | QD F | QD E | SH F | SH E | KL F | KL E | KHS F | KHS E | BS F | BS E | KTK F | KTK E | QD F | QD E | $X_{ij},Y_{ij}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| QD→ | | | | *5* | 500 | | 230 | | | | | | - | - | ←QD |
| | - | - | | | | | 140 | | 180 | | 490 | | | | |
| SH→ | | | | | 435 | | 450 | | | | | | - | - | ←SH |
| | - | - | | | 105 | *350* | 200 | | 150 | *410* | | | | | |
| KL→ | | | | | | | | *210* | 600 | | 225 | | - | - | ←KL |
| | - | - | | | 500 | | 420 | | | | | | | | |
| KHS→ | 150 | | | | | | | | 580 | | 160 | | - | - | ←KHS |
| | - | - | 500 | | | *80* | | | | | | | 180 | | |
| BS→ | 300 | *25* | 150 | *175* | | | | | | | 530 | | - | - | ←BS |
| | - | - | 455 | | | | | | | | 175 | | 405 | | |
| KTK→ | 70 | *190* | 60 | *495* | 100 | | | | | | | | - | - | ←KTK |
| | - | - | | *225* | | | | | 180 | | 550 | | | | |

Table 8: Optimal ship-slot allocations at each calling port based on average demand

| $X_{ij},Y_{ij}$ | QD F | QD E | SH F | SH E | KL F | KL E | KHS F | KHS E | BS F | BS E | KTK F | KTK E | QD F | QD E | $X_{ij},Y_{ij}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| QD→ | | | | | 320 | *320* | 162 | *162* | | | | | - | - | ←QD |
| | - | - | | | | | 166 | *166* | | | 335 | *335* | | | |
| SH→ | | | | | 363 | *363* | 327 | *327* | | | | | - | - | ←SH |
| | - | - | | | 83 | *83* | 163 | *163* | 98 | *98* | | | | | |
| KL→ | | | | | | | 338 | | 415 | *415* | 181 | *181* | - | - | ←KL |
| | - | - | | | 350 | *350* | 312 | *312* | | | | | | | |
| KHS→ | 144 | *144* | | | | | | | 403 | *403* | 111 | *111* | - | - | ←KHS |
| | - | - | 415 | *415* | | | | | | | | | 144 | *144* | |
| BS→ | 185 | *185* | 439 | *113* | | | | | | | 357 | *357* | - | - | ←BS |
| | - | - | 197 | *197* | | | | | | | 181 | *181* | 317 | *317* | |
| KTK→ | 43 | *43* | 41 | *787* | 82 | *82* | | | | | | | - | - | ←KTK |
| | - | - | | | | | | | 162 | *162* | 410 | *410* | | | |

The model is an algebraic sum of three sub-models: the fuel oil cost model is $1.64 \cdot u + 5{,}440$ per day and diesel oil cost model is $0.2066 \cdot u + 12{,}208$ per day, the ship rental model is $6.54 \cdot u + 1{,}422.52$ (2005) per day, and the cost of ship handling at calling ports is $1.95 \cdot u + 3{,}453.36$ (2005) per entry. Then the proposed formulation is solved by Mat Lab based on the GA, and results are shown as the following tables and figures.

The optimal set of calling ports with the optimal order of calling sequence based on weekly service frequency is as follows:

Qingdao ➔ Shanghai ➔ KeeLung ➔ Kaohsiung ➔ Busan ➔ Kitakyushu ➔ Qingdao.

Based on fluctuating demand, the optimal ship-size has an approximate capacity of 1,715 TEU with the maximal total profit of \$ USD 102,578.5 and maximal unit ship-slot profit of \$ USD 60 per average voyage. Based on average demand, the optimal ship-size has an approximate capacity of 2508 TEU with the maximal total profit of \$USD 42,509 and maximal unit ship-slot profit of \$ USD 17 per average voyage. Their ship-slot allocations are shown in tables 7 and 8.

When the range of fluctuating demand expands 10% and 30% respectively based on the original fluctuating demand, the optimal ship sizes with other relevant values based on fluctuation and average demand varies as shown in tables 9, 10, and 11.

Table 9: Comparison between two different demand forms based on original fluctuation data

| Original data | based on fluctuating demand | based on average demand |
|---|---|---|
| Optimal ship-size | 2×1,715 TEU | 2×2,508TEU |
| Maximal total profit per average voyage | $USD 205,157 | $USD 85,018 |
| Maximal unit ship-slot profit per average voyage | $USD 60 /TEU | $USD 17 /TEU |

Table 10: Comparison between two different demand forms based on 10% expansion of original fluctuation data

| +10% expansion of original data | based on fluctuating demand | based on average demand |
|---|---|---|
| Optimal ship-size | 2×1,826 TEU | 2×2,508TEU |
| Maximal total profit per average voyage | $USD 282,079 | $USD -340,881 |
| Maximal unit ship-slot profit per average voyage | $USD 77 /TEU | $USD -68 /TEU |

Table 11: Comparison between two different demand forms based on 30% of original fluctuation data

| +30% expansion of original data | based on fluctuating demand | based on average demand |
|---|---|---|
| Optimal ship-size | 2×1,978 TEU | 2×2,508TEU |
| Maximal total profit per average voyage | $USD 494,358 | $USD -498,917 |
| Maximal unit ship-slot profit per average voyage | $USD 125 /TEU | $USD -99 /TEU |

From the above tables, we can see that the larger the range that fluctuating demand expands, the better the optimal ship size with relevant values would be based on fluctuating demand, but the ones based on average demand would vary in reverse. Consequently, the proposed formulation based on fluctuating demand has the distinct superiority in container shipping network design to the one based on classic average demand.

As results of optimal shipping network design, the maximal container quantities handled at all the calling ports are also solved to be 810 TEU at Qingdao, 955 TEU at Shanghai, 1,035 TEU at Keelung, 890 TEU at Kaohsiung, 1,180 TEU at Busan and 1,215 TEU at Kitakyushu, respectively. Based on the maximal quantities handled and the ship-slot allocations at all the calling ports, the optimal container configurations at each calling port are obtained as shown in table 12.

Table 12: Container configurations deployed at each calling port based on fluctuating demand

| calling ports / constitutions | **QD** | **SH** | **KL** | **KHS** | **BS** | **KTK** |
|---|---|---|---|---|---|---|
| Owned quantity | 340 | 365 | 480 | 390 | 565 | 425 |
| Long-term quantity | 235 | 300 | 253 | 239 | 236 | 384 |
| Short-term quantity | 235 | 290 | 302 | 261 | 379 | 406 |
| Total quantity at port | 810 | 955 | 1035 | 890 | 1180 | 1215 |

Thus far, all the determinant factors concerning the container shipping network design have been obtained by solving the proposed formulations. Results are shown in accordance with real-world cases of container shipping route operations. They are therefore more practical and applicable than ones based on classic average demand, which existing studies have generally utilized.

# 6 Conclusions

This study addressed the optimization problem of container shipping network design based on fluctuating demand along with freight rates. Although there is a large amount of literature on ship routing and scheduling problems, few studies considered the container shipping network design problem; none of them did so based on fluctuating demand coupled with freight rates. This paper considered the influence of fluctuating demand with freight rates on ship sizing and container deployment, which are major determinant factors of network optimization structures and operations. By designing service network shipping in binary directions with handling quantities and ship-slot allocations at calling ports, the problem is formulated as a model with an objective of maximizing unit ship-slot profit in three stages that authentically represents the real-world experience of container shipping route operation. GA is used as a solution method for the problem,

Through case experiments, we have reached the following conclusions: In considering the influence of fluctuating demand, the unit ship-slot profit of optimal service network operation in binary directions is the best in comparison with ones based on fixed average demand. As a result, the problem based on fluctuating demand together with freight rates results in optimizing the smallest ship-size and corresponding container configurations which do not only gain the best voyage profit, but also largely reduce costs of asset deployment.

The proposed approach is very useful in assessing shipping network operations from both strategic and tactical viewpoints. Furthermore, it is also extremely effective in employing the unit ship-slot profit per average voyage to deal with the issue of comparison between repositioning and leasing of empty containers, and in optimizing ship-size to deal with revenue-loss control problems.

In fact, the container shipping network structure and operation should be designed not only based on fluctuating demand combined with freight rates that are determined by historical usage, but should also be designed based on projections of future fluctuating demand. A combination of these two approaches may provide an interesting topic for future research.

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