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How Many Routes Can Be Removed in Airline Network?

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Abstract We propose a mathematical programming model for reducing the number of air routes. As a case study we apply the model to Japanese domestic airline network.

Keywords airline; integer programming; network flow

1 Introduction

Since the pioneering work of O'Kelly[5], designing an airline network has been an alluring topic in operations research studies, and seemingly hub location problems are main concerns in the literature, see [1] for recent survey. In this paper a different approach is taken to the design problem in an attempt to find a compact and efficient sub-network of the original airline network.

In Fig. 1 Japanese domestic airline network in 2006 is shown over the map of Japan. Total number of regular airlines was 211 and they covers almost all the area. Recently two big Japanese airline companies announced that they started to reduce the airlines which were not profitable. Our interest in this paper is in how many links can be removed from the network with satisfying all the passengers demand by using alternative one-stop or two-stop routes. This approach is close to Jaillet et al. [2] (see also [4]), although they did not aim to reduce the number of airlines but to find the minimum cost rearrangement of airline network.

This paper is organized as follows. In Sec. 2 we give a mixed integer programming formulation for our model and some comments on similar models are followed. In Sec. 3, we apply our model to Japanese airline network and make investigations on the obtained solutions. Sec. 4 summarizes the result and gives some remarks.

2 Model Formulation

We formulate the airline network by using undirected simple network $G = (V, E)$, node set V, edge set E. A node $i \in V$ stands for airport and an edge $e \in E$ means the airline connecting both ends of e. For an edge e we denote the vertices at both ends of e by δe. so if an edge e connects vertices i and j then $\delta e = \{i, j\}$.

Let p_e be the number of passengers between airports i and j per one year, where $\{i, j\} = \delta e$. It is the sum of passengers from i to j and from j to i. The airline distance

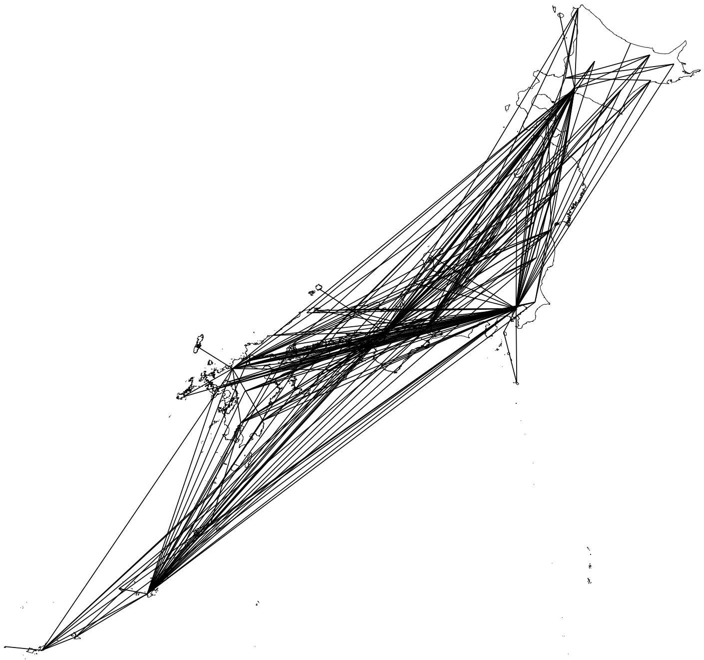

Figure 1: Japanese domestic airline in 2006

between i and j is denoted by d_e, and the number of seats available in the airline e is denoted by s_e.

Our model seeks a reduced network which satisfies all the passengers demand p_e with smaller number of airlines. The passengers of removed airlines are required to use an alternative route for their trip. An alternative route for e is a set of edges which connect both ends of e. More specific definition of alternative routes is given depending on how long a detour is allowed in the model. We consider one-stop and two-stop models in this paper, the definition of which is shown shortly. Let R_e be the set of alternative routes for an edge e and let $R = \cup_{e \in E} R_e$ be all the alternative routes. A relation between an alternative route r and an edge e is represented by so-called routing matrix $T = (t_{er})$ defined as

$$t_{er} = \begin{cases} 1, & \text{if } e \in r, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The distance of route r is given by $l_r = \sum_{e \in r} d_e$.

Let x_r be the decision variable for the number of passengers who use the route r, and z_e be 0-1 variable which is 0 if the edge e is to be removed and 1 if it remains.

We consider two models, one is minimum airline model (MAM) and another is k

airline reduction model (kARM). MAM minimize the number of airlines:

$$
\text{(MAM)} \qquad
\begin{array}{lll}
\text{min.} & \sum_{e \in E} z_e & \\
\text{s.t.} & \sum_{r \in R} t_{er} x_r \le s_e z_e, & e \in E \\
& \sum_{r \in R_e} x_r = p_e, & e \in E \\
& x_r \ge 0, \quad z_e \in \{0,1\} & r \in R, e \in E
\end{array}
\tag{2}
$$

On the other hand, kARM minimize the total traveling distance of the passengers with fixed number k of remaining airlines:

$$
(k\text{ARM}) \qquad
\begin{array}{lll}
\text{min.} & \sum_{r \in R} l_r x_r & \\
\text{s.t.} & \sum_{r \in R} t_{er} x_r \le s_e z_e, & e \in E \\
& \sum_{r \in R_e} x_r = p_e, & e \in E \\
& \sum_{e \in E} z_e = k, & \\
& x_r \ge 0, \quad z_e \in \{0,1\} & r \in R, e \in E
\end{array}
\tag{3}
$$

To fix the routing matrix T we consider two policies for choosing alternative routes. One-stop model allows non-stop flight (direct flight) and one-stop flight. We enumerate all the pair of edges $\{e_1, e_2\}$ which connect δe for each edge e to obtain the matrix T. In this case each row of matrix T contains one or two nonzero elements. Two-stop model allows two-stop flight in addition to one-stop case. In this case each row of matrix T contains at most three nonzero elements.

In both models we do not take into account of periodic airline operations, i.e. departure/arrive time, which makes our models more complicated. Nonetheless we consider our models are meaningful for analyzing network structure. Sec. 3 gives numerical examples of our models by using Japanese domestic airline network, and demonstrate several implications obtained in the analyses by using our models.

Remark: One-stop and tow-stop policies are used in Jaillet et al.[2] to study US airline network, where they dealt with directed network and obtained the minimum operation cost assignment of aircrafts. In other words their objective is to minimize the running cost of airline and the capacity of airline is chosen as the decision variable. Our formulation is close their model except a few modifications for undirected network and our objective is total number of airlines(MAM) or travelers cost (kARM) with given airlines capacity.

3 Numerical Results

Two models MAM and kARM are applied to Japanese domestic airline network shown in Fig. 1. The data are taken from [3] and contain the number of passengers p_e of each airline for one year in 2006 and the airline distance d_e. We first solve MAM to find the minimum number of airlines, say $k_{\min}$, then solve kARM for $k \ge k_{\min}$.

We consider one-stop and two-stop model in MAM, and call the MAM with one-stop model MAM1 and two-top model MAM2. The result is shown in the Table 1. If we allow one-stop route for the trip, 115 airlines are necessary for satisfying all the passengers' trip among existing 211 airlines. and only 90 airlines are necessary for all the trip, if we allow two-stop route.

Table 1: MAM1 and MAM2 results for Japanese domestic airline in 2006

Num. of airlines	$k_{\min}$ by MAM1	$k_{\min}$ by MAM2
211	115	90

After obtaining $k_{\min}$, we solve one-stop kARM, and two-stop kARM, for $k > k_{\min}$. Optimal values are shown in Fig.2. They are scaled by dividing the original total traveling distance $\sum_e p_e d_e$. In one-stop kARM, the increase of total traveling distance is less

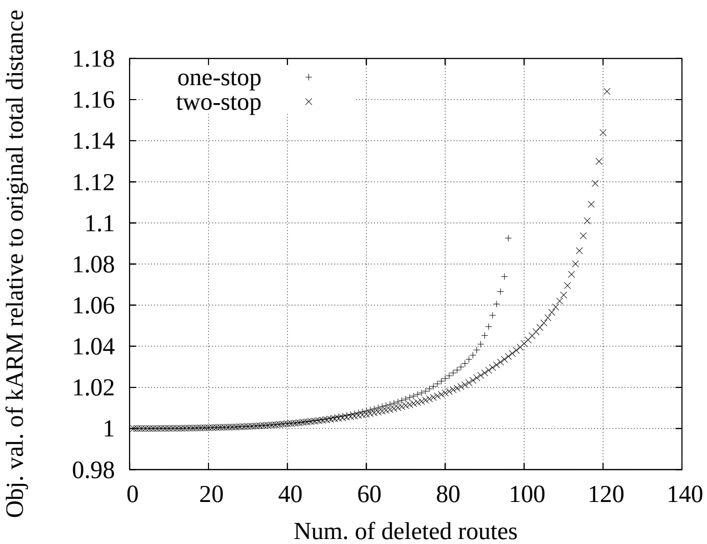

Figure 2: One-stop and two-stop kARM result for $90 \leq k \leq 210$. Horizontal axis shows the number of delete routes $k - k_{\min}$. Vertical axis shows the objective function value of kARM relative to original total distance.

than 1% when the number of deleted airlines is under 60, and it is less than 10% at the maximum deletion 96. Two-stop kARM has similar value of total traveling distance with one-stop up to deletion number 60, and when the number of deleted airline become more than 60, the total distance shows slower increase than one-stop kARM. At the deletion number 96 two-stop kARM exhibits a increase of approximately 4% in total distance, and even at the maximum deletion number 121 of airlines it shows only less than 20% increase.

Another analysis in the numerical study is on the "hub possibility" of several airports in Japan. We calculate the number of passengers who use the several airports. Fig. 3 shows the results for some main airports in Japan, which are HND(Haneda), FUK(Fukuoka), NGO(Nagoya), CTS(Chitose), and ITM(Itami). In both one-stop and two-stop model Haneda increase the number of passengers as deleted airline increases.

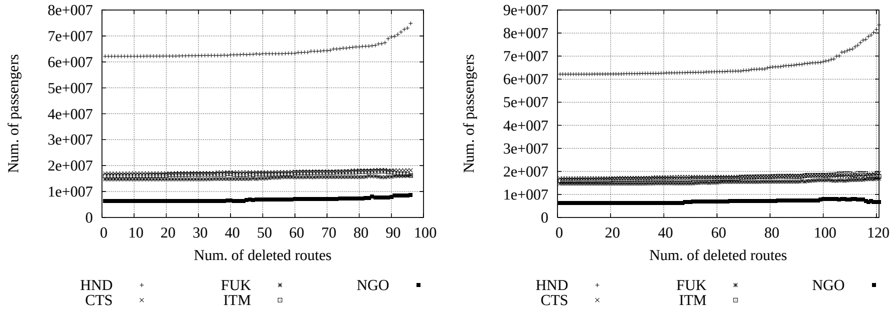

Figure 3: The number of passengers going through main airports in Japan. Left: one-stop case, Right: two-stop case.

Since Haneda has big share and Fig. 3 is inconvenient for the analysis of other airports, the number of other airport passengers are re-depicted in Fig. 4. In one-stop case every airport except Itami shows the increase of passengers as the number of the deleted airlines increases. Only Itami shows the increase of passenger up to the number of deletion 90, and after 90 route are deleted it shows the decrease of passengers. In two-stop case Itami and Nagoya shows the decrease of passengers after 110 routes are deleted, while other airport increase the passengers.

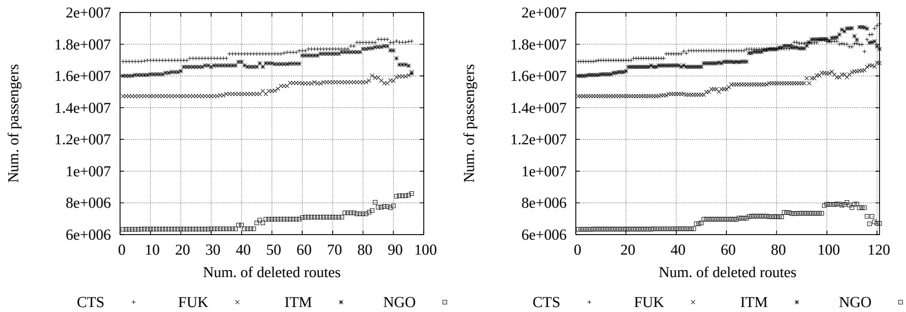

Figure 4: The number of passengers going through main airports in Japan (HND excluded). Left: one-stop case, Right: two-stop case.

The number of survived airlines in one-stop model is 115, which are shown in the left of Fig. 5. The right of Fig. 5 shows survived airlines in two-stop model, the number of which is 90. In both maps we can see dense lines on the long area lying east and west, where Japanese economy and industry are concentrated. Most of other lines connects this area and north or south part of Japan, although they scatter more sparsely than Fig. 1. Comparison of Fig. 5 and Fig. 1 tells us that removed are those lines connecting between north region and west region, or north and south region (it contains many islands).

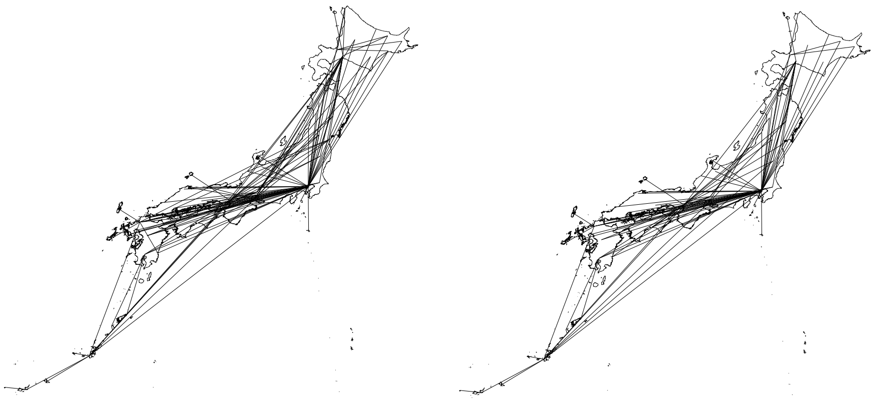

Figure 5: Result for one-stop(left) and two-stop(right) MAM in 2006

4 Concluding Remarks

We applied airline reduction models to Japanese domestic airline network to obtain several findings. Firstly surprisingly small number of airlines are sufficient to meet the passengers demands. We know almost half of total airlines can be reduced in our model. Secondly the total cost incurred by using one-stop or two-stop flights is relatively small. The cost is measured by total traveling distance and increases by 20% when maximum number of airlines are deleted. There are many small capacity airlines in Japanese air-network, and they are often connected to a big hub airport. Such observation seemingly gives an explanation for our results.

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