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# Analysis of Energy Consumption in 159 Countries by Triangular Pyramid Diagram

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**Abstract** We use a triangular diagram (also known as a ternary diagram) and triangular pyramid diagram (TPD) to analyze energy consumption data by energy type. To consider the energy problem, it is important to understand the energy type breakdown of the energy consumption. A triangular diagram is effective for the analysis of data of three different energy types. We can classify energy into three kinds and present the data in a triangular diagram. However, the type of energy is often classified into four types, in which case a triangular diagram is insufficient. Accordingly, we developed a 3-dimensional TPD. We describe the formulation of this TPD, and use it to analyze the energy consumption data of 159 countries by energy type. Though a stereogram is used in this paper, we demonstrate the triangular pyramid diagram by animation at live presentations.

**Keywords** Triangular Pyramid Diagram, Triangular Diagram, Energy Consumptions

## 1 Introduction

The problems of global warming and the depletion of worldwide energy resources are very serious. Therefore, it is important to understand the state of energy consumption of every country. To analyze the energy problem and $CO_2$ emissions, Akimoto et al.[1] and Fujii et al.[2] developed a global energy system model. There have been a number of attempts to develop global energy models. therefore, techniques for grasping the energy data are valuable.

In this paper, we analyze the energy consumption of countries according to energy types using a triangular diagram and the triangular pyramid diagram. Using these diagrams, we examine the state of each country and the future trend of energy consumption. The United Nations has assembled complete energy consumption data for 159 countries. The energy consumed by these 159 countries accounts for 99.9% of the global energy consumption (9 billion 53 million / 9 billion 59 million (tons, oil conversion, surveyed by the United Nations, 2004)). In a same way, their total population accounts for 98.8% (6 billion 438 million / 6 billion 515 million persons) of the world's population. We chose object countries based on three conditions. The

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conditions are: (1) the 2004 GDP estimate is valid, (2) the 2004 population estimate is valid and (3) the annual consumption is more than 200,000t (more than 1/10000 of that of United States, where consumption has been the highest).

The triangular diagram is effective for the analysis of the data composed of three kinds of ratios, and it is widely used, particularly in geology, and physical chemistry and metallurgy (Howarth, [4]). It seems a simple idea to enhance the triangular diagram to three dimensions for the analysis of the data made by four kinds of ratios, and this idea was applied by Ogawa [3] to analyze electoral districts. However, this tool is not frequently used because of the difficulty of presentation and convenience. Because the energy data is currently classified into four (solid, liquid, gas, and electricity), the triangular pyramid diagram is effective for its graphic representation. In this research, triangular pyramid diagrams are presented by stereogram. Furthermore, other indexes such as consumption per capita are shown by spherical with volumetric representation in the triangular pyramid diagram.

## 2 The World Energy Consumption Data

The data are defined as follows. (1) Energy for commerce; (2)Consumption is defined as [Production] + [import] - [export] - [consumption under transportation] ± [stock]; (3) Consumption in the chemical industry contains only the fuel; (4) The transportation consumption includes only transportation in inland traffic and the neighboring waters; (5) As mentioned above, energy use is classified into four types: solid (coal, briquettes, oil shale, bituminous sands, etc.), liquid (crude petroleum, alcohol, natural gas liquids, etc.), gas (natural gas, biogas, etc.), and electricity (electricity from hydro, nuclear, solar, wind, etc.).

Countries in the top 5 or bottom 5 in terms of population, GDP or amount of energy of consumption in 2004 are shown in Table 1. GDP was calculated by the US dollar fixed to the rate in 1990. Among the countries, there exist enormous demographic and economic differences; the maximum difference is about 13000-fold for population, and 15000-fold for GDP.

## 3 Analysis of Energy Consumption Balance by Triangular Diagram

A triangular diagram is a particular type of graph which consisting of an equilateral triangle in which a given plotted point represents properties and constraints by $x_1+x_2+x_3=1$. Let us define the data vector $\mathbf{x}=(x_1, x_2, x_3)$, the apex vector of the triangle as $f_1$, $f_2$ and $f_3$, and the position on the diagram as $\mathbf{P}=(p_1,p_2)$.

Then the transformation equation is set up as follows:

$$\mathbf{p} = \sum_{i=1}^{3} x_i \mathbf{f}_i \tag{1}$$

If the gravity point of a triangular diagram is set to origin $\mathbf{x_0}$ as (1/3, 1/3, 1/3), the transformation of data to a position on the triangular diagram is as follows.

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = x_1 \begin{bmatrix} -1/2 \\ -1/2\sqrt{3} \end{bmatrix} + x_2 \begin{bmatrix} 1/2 \\ -1/2\sqrt{3} \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1/\sqrt{3} \end{bmatrix}$$
$$= \begin{pmatrix} -1/2 & 1/2 & 0 \\ -1/2\sqrt{3} & -1/2\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (2)$$

Table 1: Top 5 and bottom 5 countries in terms of energy consumption, GDP, and population.

| | pop(×1000) | | | GDP(×＄1000000) | | | Oil Equip kt | | |
|---|---|---|---|---|---|---|---|---|---|
| **Country or Area** | **1984** | **1994** | **2004** | **1984** | **1994** | **2004** | **1984** | **1994** | **2004** |
| United States | 240552 | 267362 | 296844 | 4711846 | 6345161 | 8734868 | 1605419 | 1881245 | 2051914 |
| China | 1052132 | 1201712 | 1304983 | 231080 | 615815 | 1477367 | 471072 | 763155 | 1260182 |
| Japan | 120141 | 125119 | 127798 | 2255762 | 3168378 | 3530123 | 315907 | 394406 | 441823 |
| India | 754051 | 935455 | 1116985 | 226061 | 387172 | 702558 | 123677 | 239291 | 364924 |
| Germany | 77653 | 81314 | 82628 | 1410683 | 1867940 | 2160134 | 328901 | 297191 | 294756 |
| Italy | 56589 | 57193 | 58475 | 946365 | 1174179 | 1370285 | 136854 | 159079 | 179911 |
| France | 54993 | 57952 | 60624 | 1045994 | 1299962 | 1623420 | 155964 | 152726 | 174227 |
| Brazil | 133257 | 159197 | 184318 | 370693 | 488664 | 619578 | 69453 | 107099 | 148337 |
| Indonesia | 164132 | 194540 | 223225 | 87813 | 169357 | 226595 | 36096 | 75521 | 101645 |
| Netherlands Antilles | 180 | 192 | 184 | 1905 | 2294 | 2313 | 1799 | 852 | 2250 |
| New Caledonia | 153 | 189 | 230 | 1494 | 2751 | 3008 | 401 | 542 | 808 |
| Suriname | 377 | 413 | 450 | 412 | 453 | 603 | 564 | 592 | 716 |
| Guyana | 758 | 737 | 739 | 436 | 532 | 666 | 460 | 437 | 479 |
| Belize | 159 | 208 | 269 | 251 | 481 | 769 | 58 | 128 | 279 |
| Sierra Leone | 3495 | 4129 | 5390 | 920 | 817 | 1087 | 187 | 128 | 252 |
| Aruba | 64 | 80 | 101 | 447 | 1124 | 1431 | 0 | 274 | 251 |
| Eritrea | 2735 | 3186 | 4354 | 0 | 1169 | 1445 | 0 | 53 | 242 |
| Maldives | 178 | 242 | 291 | 116 | 278 | 584 | 19 | 73 | 240 |
| French Polynesia | 170 | 212 | 252 | 2221 | 3129 | 4203 | 152 | 190 | 230 |
| Mali | 6638 | 8508 | 11265 | 1849 | 2713 | 4754 | 155 | 171 | 210 |

○:Europe, □:CIS, ●: (South) East Asia & Oceania, ■: West & South Asia, ●:Africa, ■:America

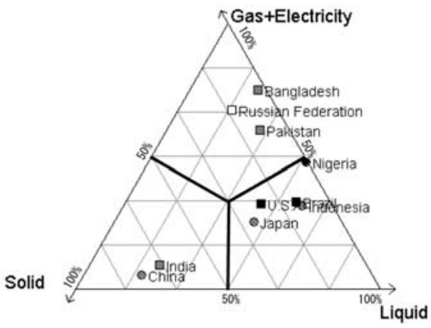


Figure 1: A triangular diagram of energy data with *superior area* (10 countries)

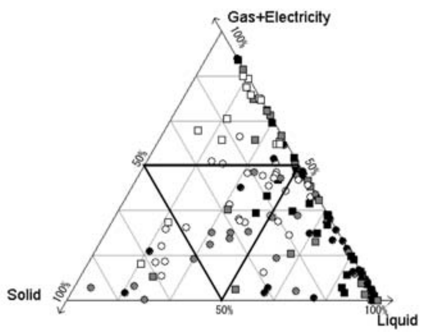


Figure 2: A triangular diagram of energy data with *balanced area* (159 countries)

Figure 1 shows the balance of the energy consumption of the 10 most populous countries in 2004 by triangular diagram. The upper apex denotes Gas+ Electricity=

100%, the lower left apex denotes solid=100% and the lower right apex denotes liquid=100%. We show energy with the largest ratio according to Y shape mark. The center of the symbol Y is equal to the center of gravity. Thus, three kite-shaped quadrangles created by the Y shape and the triangle show the *superior area* where the energy use is the largest. Japan mainly consumes liquid energy, while the Russian Federation consumes mainly Gas + Electricity. China and India mainly consume solid energy. The reverse triangle at the center in Figure 2 denotes the *balanced area*, which encloses countries that consumes two or more kinds of energy. Symbols of each circle are different according to the region of the country to which it belongs. For the countries in the *balanced area*, no type energy exceeds 50%. By contrast, the three triangles outside the *balanced area* are *mono energy areas* in which an energy ratio exceeds 50%.

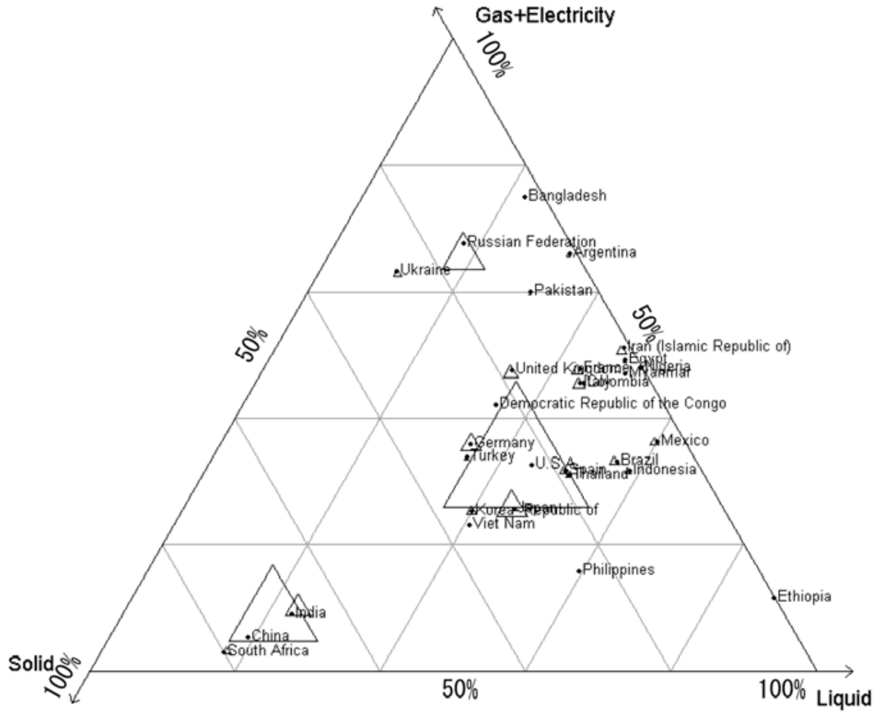

Figure 3: A triangular diagram of energy data with the total consumption triangle of every country

The energy balance and consumption in the 30 most populous countries in 2004 is shown in Figure 3. In this figure, the length of edges of a small triangle is proportional to the total amount of energy consumed in each country. And, the total length of the edges of all the countries equals the total consumption amount. The position of the point in each small triangle shows the energy balance both in the triangular diagram (big triangle) and in each small triangle. Consumption is the greatest in the United States, and next greatest in China, Russia, and Japan, in that order.

We can draw such small triangles by the following procedure: Let $\lambda \in [0,1]$ as the consumption ratio of the object country which is defined by [consumption amount of the country] / [world consumption amount]. The apex vector of the small triangle is written as follows:

$$\mathbf{g_i} = \lambda \mathbf{f_i} + (1-\lambda)\mathbf{p_i} \quad \mathbf{i} = 1,2,3 \tag{3}$$

where $\mathbf{g_i}$ are apex vectors of small triangles.

# 4 The Triangular Pyramid Diagram

Four kinds of energy data can be displayed in the 3D space. The triangular diagram is amplified and data are displayed on a triangular pyramid. Representation is in 2D space; hence we need devices for presentation. Let us define the data vector $\mathbf{x}=(x_0, x_1, x_2, x_3)$ constrained by $x_0+x_1+x_2+x_3=1$. We also define apex vectors of the triangles as $f_0$, $f_1$, $f_2$ and $f_3$, and a position on the diagram as $\mathbf{q}=(q_1, q_2, q_3)$. Then the transformation equation is

$$\mathbf{q} = \sum_{i=0}^{3} x_i \mathbf{f}_i \tag{3}$$

If the gravity point of a Triangular Pyramid Diagram (TPD) is set to origin $\mathbf{x_0}$ as (1/4, 1/4, 1/4, 1/4), the transformation of data to position on the TPD is as follows.

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = x_0 \begin{bmatrix} -1/2\sqrt{3} \\ -1/2 \\ -1/2\sqrt{6} \end{bmatrix} + x_1 \begin{bmatrix} -1/2\sqrt{3} \\ 1/2 \\ -1/2\sqrt{6} \end{bmatrix} + x_2 \begin{bmatrix} 1/\sqrt{3} \\ 0 \\ -1/2\sqrt{6} \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 3/2\sqrt{6} \end{bmatrix}$$
$$= \begin{pmatrix} -1/2\sqrt{3} & -1/2\sqrt{3} & 1/\sqrt{3} & 0 \\ -1/2 & 1/2 & 0 & 0 \\ -1/2\sqrt{6} & -1/2\sqrt{6} & -1/2\sqrt{6} & 3/2\sqrt{6} \end{pmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{4}$$

Figure 4 shows an image of a TPD by vector **f**. If one datum equals zero, a triangular diagram is drawn on the surface of a TPD. The *superior area* in the TPD is a hexahedron by six kite-shaped quadrilaterals of 2 varieties, as shown in Figure 5 with the 20 most populous Asian countries (TPD stereogram). And, the *balanced area* of a TPD is a regular octahedron as shown in Figure 6 with the 10 most populous countries. In Figure 6, we can see that the United States and Japan are in the *balanced area*. Since every person is not necessarily good at binocular vision, animation is used in the live presentation.

# 5 Analysis of Energy Consumption Balance by TPD

Figure 7 shows the energy sharing ratio using the data of 159 countries in 2004. The apex of electricity energy is located in the back in Figure 7. We can find the following by seeing Figure 7. The country at a position that is the nearest center of gravity is Kyrgyzstan, for which the energy ratio vector **x** (solid, liquid, gas, electricity) is equals to (20%, 20%, 27%, 33%). There are 36 countries in the balanced area. African nations tend to depend on liquid-type energy, and CIS nations tend to depend on gas. Most of west Asia and South Asia nations tend to depend only on gas and liquid, and line up on the edge of liquid-gas. Turkey is located at the center of the triangle of liquid, gas, and solid. Similarly, European nations relatively distribute other three kinds of energy except the electricity. As an exception, European nations depending chiefly on solids are the nations of the former Yugoslavia. In countries where the ratio of electric power is especially high, the consumption gross weight is often low, as in Mozambique, Zambia. In North Korea, China, India, Mongolia and South Africa, dependence on solid energy is especially high.

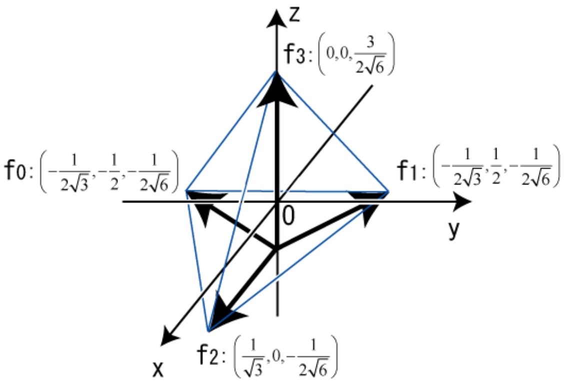


Figure 4: Vectors of TPD

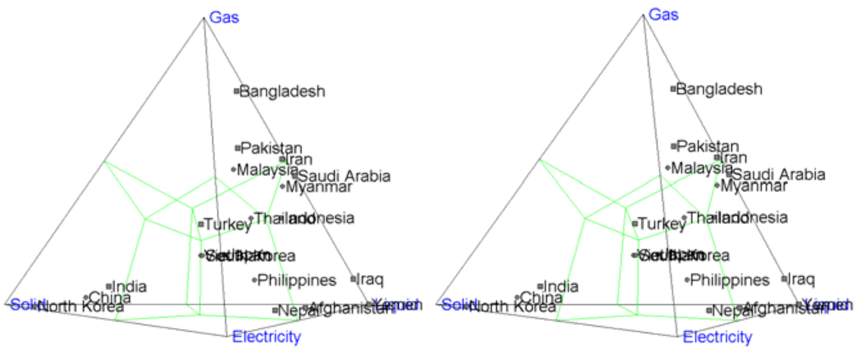


Figure 5: Twenty most populous Asian countries on a TPD and the *superior area* (Stereogram, the apex of Electricity is located in the back)

Figure 8 shows fifteen countries on the TPD with consumption per capita; which represent the sum-set of ten most populous nations and the ten nations with the largest energy consumption. European nations which have large per capita consumption are located in the *balanced area*. Affluent nations in general tend to be located in the *balanced area*.

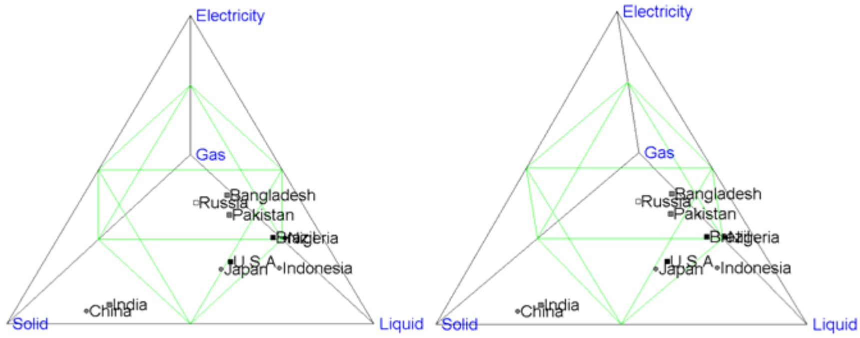


Figure 6: Ten most populous countries on a TPD and the *balanced area* (Stereogram, the apex of Gas is located in the back)

◦:Europe, ▫:CIS, •: (South) East Asia & Oceania, ▪: West & South Asia, •:Africa, ▪:America.

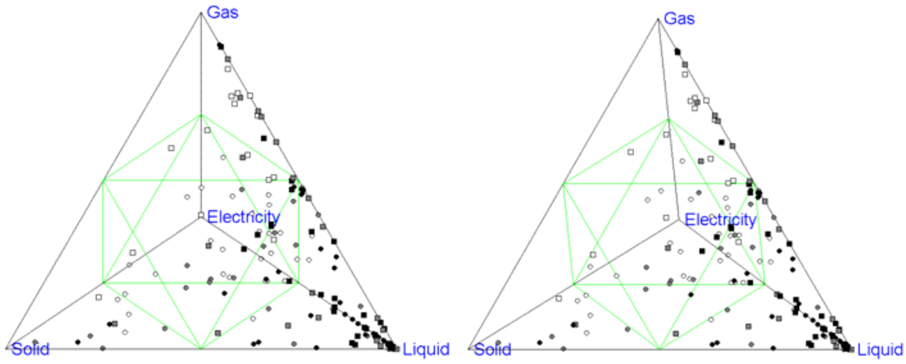


Figure 7: The 159 countries on a TPD and the balanced area (Stereogram, the apex of electricity is located in the back).

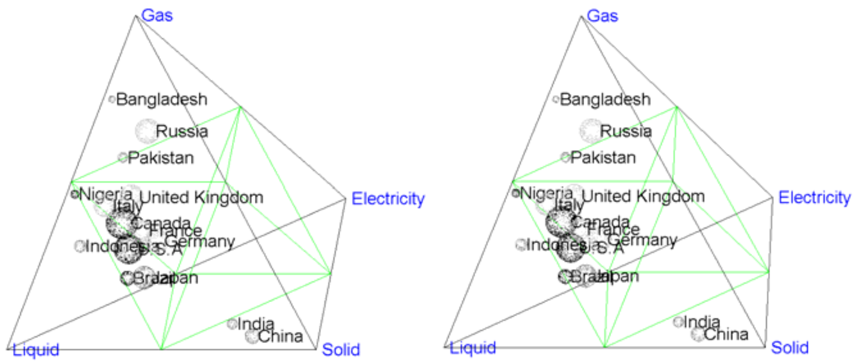


Figure 8: Fifteen countries on a TPD with consumption per capita (Stereogram, the apex of Electricity is located in the front)

## 6 Conclusion

Analysis of the energy consumption of countries with sharing ratios by triangular diagram and TPD has been described. Visualization by our method may be very effective in the analysis of data which is classified into four types. Consideration of the cause of the current energy sharing ratios and future trends should be researched as the next stage.

The software of TPD and the triangular diagram is made available to users on condition that it is not used for commercial purpose, and also that suitable acknowledgement will be made. If you want, please contact Kotoh by e-mail.

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