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Scheduling with Rejection to Minimize the Total Weighted Completion Time*

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Abstract In this paper, we address the scheduling problem with rejection in which we can choose a subset of jobs to process. Choosing not to process any job incurs a corresponding penalty. We consider the following problem for the first time: scheduling with rejection to minimize the total weighted completion time with the constraint of total penalties on identical parallel machines, where the number of identical parallel machines is constant. We show that it is NP-hard and design a pseudo-polynomial time algorithm as well as an FPTAS through dynamic programming.

Keywords Scheduling, Rejection, Identical parallel machines, Approximation algorithm

1 Introduction

In this paper, we study the scheduling problem with rejection in which we can choose a subset of jobs to process on identical parallel machines to minimize the total weighted completion time. In the classical scheduling, it is always assumed that for any job, we have to process it. However, in the real world, things may be more flexible and we can make a higher-level decision, i.e., we can break the constrain by rejecting a job. It's not hard for the readers to find examples in the industrial and commercial fields to justify this breaking. To reject a job, of course, we should pay a corresponding penalty.

In the rest of this paper, we will denote by $\{J_1, J_2, \cdots, J_n\}$ a list of given jobs. We write SR as abbreviation of the scheduling problem with rejection . In the SR model, each job $J_j (1 \leq j \leq n)$ is characterized by a double (p_j, e_j), where p_j is its processing time if we choose to process it(or accept it) and e_j the penalty we pay if we reject it. We also denote by TP the total penalties of the rejected jobs in SR.

SR is in essence bi-criteria, thus there are the following four models for us to study:

(P1) To minimize $F_1 + F_2$;

(P2) To minimize F_1 subject to $F_2 \leq a$;

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(P3) To minimize F_2 subject to $F_1 \leq b$;

(P4) To identify the set of Pareto-optimal points for (F_1, F_2).

Where F_1 is the original objective function, and F_2 is TP . In the objective function field of the notation of Graham *et al.* [1], we write the above four model as $F_1 + F_2$, F_1/F_2, F_2/F_1 and (F_1, F_2), respectively. We use rej in the job environment field to characterize SR.

2 Previous related work and our contributions

As to the makespan criterion, Bartal *et al.* [2] studied the off-line version as well as the on-line version on identical parallel machines; Seiden [3] concentrated on the preemptive version and for the uniform machines variant, He *et al.* [4] presented the best possible on-line algorithms for the two machine case and a special three machine case; For the preemptive off-line variant on unrelated parallel machines, Hoogeven *et al.* [5] proved that this problem is APX-hard and designed a 1.58-approximation algorithm. As to the total weighted completion time criterion, Engels *et al.* [6] addressed the off-line version and Epstein *et al.* [7] the on-line version for a unit-weight-unit-processing-time special case. Sengupta [8] also considered the maximum lateness/tardiness criterion.

We note that all the researches cited above aims at the P1 model. Cao and Zhang [9] showed that the P2 model for $1|rej|\sum w_jC_j/TP$ is NP-hard and designed an FP-TAS(Fully Polynomial Time Approximation Scheme) for it. In this paper, we address for the first time the following problem : scheduling with rejection to minimize the total weighted completion time with the constraint of total penalties on identical parallel machines, namely, the P2 model for $P_m|rej|\sum w_jC_j$, where the number of identical parallel machines is constant. We show that it is NP-hard and design a pseudo-polynomial time algorithm as well as an FPTAS. Our approach is dynamic programming and the so called trimming the state space technique.

3 Dynamic programming algorithm and FPTAS

Lemma 1 [9] *$1|rej|\sum w_jC_j/TP$ is NP-hard.*

Since $P_m|rej|\sum w_jC_j/TP$ takes $1|rej|\sum w_jC_j/TP$ as a special case, it is NP-hard.

In the following, we will design a pseudo-polynomial time dynamic programming algorithm and an FPTAS for it.

We are given a list of jobs $\{J_j = (p_j, w_j, e_j) : 1 \leq j \leq n\}$ and a set of identical parallel machines $\{M_1, M_2, \cdots, M_m\}$, where m is a constant, each job has to be processed by exactly one machine. The given threshold for TP is E. We will find a schedule with the minimum total weighted completion times whose TP is at most E.

We suppose that all the jobs have been indexed in non-decreasing order of p_j/w_j. For any partial schedule for jobs $J_1, J_2, \cdots, J_j$, let P^i be the total processing times, and A^i be the the total weighted completion times on machine $M_i (1 \leq i \leq m)$. Let $P = (P^1, P^2, \cdots, P^m)^T, A = (A_1, A_2, \cdots, A_m)^T$, which are $m-$ dimensional vectors, we say that its state is (j, P, A).

Let $f(j, P, A)$ be the minimum TP of partial schedules for jobs $J_1, J_2, \cdots, J_j$, whose total processing times are P^i, $0 \leq P^i \leq \sum_{j=1}^{n} p_j$, and total weighted completion times are A^i on machine $M_i (1 \leq i \leq m)$, Let E_v is a $m-$dimensional unit vector and the vth component of which is 1, thus we have:

$$f(1,P,A) = \begin{cases} 0 & if\ PE_v = p_1 \quad and \quad AE_v = w_1 p_1 \\ e_1 & if\ PE_v = 0 \quad and \quad AE_v = 0 \\ +\infty & \text{otherwise} \end{cases}$$

$$f(j,P,A) = \min\{f(j-1,P-p_jE_v,A-w_jPE_v), f(j-1,P,A)+e_jE_v\}$$

It's straightforward that the time complexity is $O(n(nP_{\max})^m(n^2P_{\max}w_{\max})^m)$, which is pseudo-polynomial.

In order to get a $(1+\varepsilon)-$approximation in polynomial time, similarly, we need to use the trimming the state space technique as we have done for $P_m|dm|C_{max}/TPC$ [10].

The only difference is that for any state vector (j,P,A), we may using the stretching technique twice: if $P^i(1 \leq i \leq m)$ is not an integer power of $1+\varepsilon_0$, where $\varepsilon_0 = \varepsilon/(4n)$, we should first stretch p_j a little such that P^i is of this form and then stretch w_j such that $A_i(1 \leq i \leq m)$ is also of this form. For the case P^i is already an integer power of $1+\varepsilon_0$, which maybe because that J_j is rejected in this state, we merely have to stretch w_j. After the two steps of operations, A_i may eventually be enlarged to at most $(1+\varepsilon_0)^2$ times the original value. It's still easy to calculate that the corresponding running time is $O(n((1/\varepsilon)n\log(nP_{\max}))^m((1/\varepsilon)n\log(n^2P_{max}w_{max}))^m)$. The details are left to the interested readers.

Theorem 2 *$P_m|rej|\sum w_jC_j/TP$ admits an FPTAS.*

4 Conclusion and remarks

In this paper we have discussed the scheduling problem with rejection. We address for the first the P2 model for scheduling with rejection to minimize the total weighted completion time on identical parallel machines, where the number of identical parallel machines is constant. We show that it is NP-hard and design a pseudo-polynomial time algorithm as well as an FPTAS through dynamic programming. For the P4 model for any scheduling problem with rejection is of great interest, further discuss is still needed.

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