EMPLOYING POST-DEA METHOD IN BUDGET MANAGEMENT OF HEALTH CARE SECTORS

Xiao-Ya Li

Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
xyli@amss.ac.cn

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Abstract

An increasing attention has been paid to efficiency analysis in the health care area. Among the existing efficiency assessment techniques, data envelopment analysis (DEA) plays an important role in a wide range of applications as for measuring the relative efficiency of different health care sectors. This paper focuses on a second stage of the analysis, which is operated after efficiency evaluation and called as post-DEA stage, and mainly cope with the budget management problems such as budget allocation and budget prediction. In the post-DEA stage, a comprehensive DEA based techniques are adopted to make the budget prediction and allocation based on the outcome of the first stage. A framework of budget management from macro aspect is built, together with corresponding resource allocation and prediction models proposed from micro aspect. The effect of the post-DEA method is illustrated by a numerical example with considering 10 hospitals, and with considering elasticity in post-DEA method, the outcome leads to an efficiency incentive effect in budget management.

1 Introduction

With the health care sectors accounting for a sizeable proportion of national expenditures, the pursuit of efficiency has become a central objective of policy makers within most health systems. The international concern was crystallized in the World Health Report 2000 produced by the World Health Organization[1][2], which put up with the determinants and measurement of health system efficiency. Policy makers need to pay more attention on the objectives of their health systems, on how achievement might be measured, and on whether resources are being deployed efficiently[3].

Since the national and local governments have a natural requirement to ensure that finance is deployed effectively, to find the methodologies which offer insights into efficiency have attracted the interest of policy makers. One of the most commonly used tools for efficiency measurement is data envelopment analysis (DEA)[4], which obtains the efficiency result via a nonparametric, mathematical linear programming framework. DEA has been used in numerous sectors, including health care in which its first application dates back to the 1970s, and has become the dominant approach to efficiency measurement in health care and in many other sectors of the economy[5].

DEA traditionally uses the terminology of a decision making unit (DMU) for each of the unit of analysis, a term coined by Charnes, Cooper, and Rhodes in their seminal paper [4]. Since in practice, the production function for efficiency measure is too complex to get, DEA literature copes with this by estimating the function from empirical data. Efficiency in DEA is defined as the ratio of a weighted sum of outputs divided by a weighted sum of its inputs. CCR model which is built on the assumption of constant returns to scale of activities is the first model of DEA displayed as follows.

$$
\begin{array}{lll}
(CCR) & \mathbf{min} & \theta \\
 & \mathbf{s.t.} & X\lambda \le \theta x_o \\
 & & Y\lambda \ge y_o \\
 & & \lambda \ge 0
\end{array} \tag{1}
$$

In formula (1), θ stands for the technical efficiency score of DMU. Since the very beginning of DEA studies, various extension of the CCR model have been proposed, among which the BCC model is representative. BCC model take into account of various returns to scale, and categories various DMU into four types: increasing, constant, decreasing returns to scale and the congestion. Formula (2) also provides scale efficiency score of DMU.

$$
\begin{array}{lll}
(BCC) & \mathbf{min} & \theta \\
 & \mathbf{s.t.} & X\lambda \le \theta x_o \\
 & & Y\lambda \ge y_o \\
 & & e\lambda = 1 \\
 & & \lambda \ge 0
\end{array} \tag{2}
$$

In the first stage, DEA models provide valuable information to policy makers about the efficiency situation of health care sectors. The next question is that: how to employ the efficiency evaluation results into the following decision making process? Such decision making problems exist in a wide range, such as cross-evaluation, cluster analysis[6], elasticity analysis[7], and resource allocation about how to allocate constraint resources to achieve maximal efficiency. Then a second stage is derived about post-DEA models. During the second stage, all the above models are considered in the so called post-DEA framework, which help policy makers manage expenditures by using efficiency evaluation results from the first stage. New developments in performance forecasting and resource estimation come into important research topics in post-DEA field.

Post-DEA is a set of models which stands for operations after the evaluation stage, rather than a particular model. In this paper, post-DEA mainly contains inverse DEA model, common weights of DEA, elasticity model of DEA, and resource allocation model based on DEA. Zhang and Cui[8] developed a project evaluation system which firstly extend DEA model to inverse DEA field to solve the resource allocation problem. They proposed two kinds of resource allocation problem: resource allocation problem and investment analysis problem, the former one was modeled to a one-dimensional parameter problem. In [9], the inverse DEA problem is transformed into and solved as a multi-objective programming problem. In [10], a common algorithm is provided to solve the inverse DEA problem. In [11], a comprehensive resource allocation framework is built. In [12], resource allocation model based on DEA and game theory are combined together to design the water resource allocation mechanism. In [7], a post-DEA analysis with introducing elasticity is proposed. In [13], an improved inverse DEA model allowing the efficiency score being changed is put up with. Besides, post-DEA models is assumed not only to consider models in DEA field, but also to include models of optimization[11], stochastic regression[6] and game theory[12], etc.

The rest of this paper is organized as follows. Section 2 poses the framework of how to adopt certain models in different stage of decision making, following with a brief explanation of post-DEA models used in this paper. Section 3 provides complementary discussion of post-DEA applications from macro and micro level respectively, then employs the post-DEA framework into a numerical example with 10 hospitals. Conclusion is displayed in section 4, which indicates the advantages and disadvantages of post-DEA method, meanwhile depicts the application potential of the post-DEA method.

2 Methodology

We firstly portrait the framework of post-DEA methodology in figure 1. DEA models locates in the first stage, which mainly focuses on efficiency evaluation. The post-DEA stage is divided into two parts as macro-level models and micro-level models. The centralized resource allocation part mostly cope with decision problems of central managers who control all the resources and budgets, and intend to distribute them under efficient criterion to all the DMUs. The solution of such problem incorporates various tools such as multiple objective programming, game theory, parametric programming, returns to scale analysis and common weights for full ranking models. Forecasting part mainly copes with decision problems of particular DMU who is making decisions from its own aspect and intends to maximize its potential increments. The solution of such problems incorporate models such as inverse DEA models, common weights, and elasticity analysis. Dynamic management and long-term prediction is the further stage intends to deal with resource allocation and forecasting problems under changing environment. As it maintains too much complexity and technical details beyond our consideration in this paper, but it must be the future interest of DEA method.

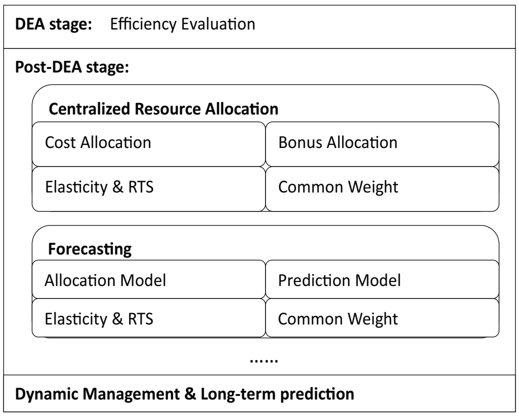

Figure 1: Framework of Post-DEA models.

2.1 Common weights of DEA model

Using DEA method to solve the common weights, the idea is firstly proposed by cook et al[14]. This paper employs the following common weights model. We get rid of the constraints $\bar{\theta}_i \le 1$ constraints in the original common weights shown in paper [15], since we modify the assumption that the production frontier is unchanged. In this paper, we choose $p = 2$ in formula (3).

$$
\begin{aligned}
(\theta_{CW}) \quad \textbf{min} \quad & \sum_{i=1}^{n} |\theta_i^* - \bar{\theta}_i|^p \\
\textbf{s.t.} \quad & \frac{\sum_{j=1}^{s} u_j y_{ij}}{\sum_{j=1}^{m} v_j x_{ij}} = \bar{\theta}_i, \quad i = 1, 2, \ldots, n \\
& u_i > 0, \quad i = 1, 2, \ldots, s \qquad (3) \\
& v_i > 0, \quad i = 1, 2, \ldots, m
\end{aligned}
$$

The common weights dependent on the efficiency score computed by DEA model, which reaches the optimal relative efficiency of the DMU. Thus the common weights make all the DMUs arrive their optimal efficiency in a compromise way[10].

2.2 Extra resource allocation algorithm based on DEA

The extra resource allocation problem is described as follows[8]. *Suppose there are some extra resource which can be given to all or only a part of DMUs, and if we want the allocation to be most beneficial to the whole system, how the extra resource should be distributed.* This extra resource allocation problem can be generally found in practice as bonus allocation. For single input system or single output system, the extra resource is allocated according to criteria index of $\theta * x$ or y. For multi-input multi-output system, the optimal solution of extra resource problem is hard to obtain, in this case, we can employ common weights to get a weighted output value or a weighted input value, and thus get a most compromise solution[10].

2.3 Inverse DEA model

Another resource allocation problem is solved by a kind of inverse DEA method. In inverse DEA framework, RAM (Resource Allocation Model) and IPM (Investment Prediction Model) are mainly investigated.

An investment analysis problem in [10] is abstracted as follows: a set of DMUs have efficiency indices $\theta_1, \cdots, \theta_n$. Assign an increment, $\Delta \mathbf{x}^l \geq \mathbf{0}$, to the input of S_l which has efficiency score θ_l. Find the "largest" additional resources, $\Delta \mathbf{y}^l$, to the output of S_l such that the resulted status of S_l remains its efficiency score unchanged. The IPM is to find the "largest" solution $\Delta \mathbf{y}^l$ such that the optimal value $\theta_{n+1} = \theta_l$, where θ_l is given by (1). Cui et.al. proposed an algorithm for solving the above model which is equivalent to the following multi-objective programming(MOP).

$$
\begin{aligned}
(IPM) \quad \mathbf{max} \quad & (\Delta y_{1o}, \cdots, \Delta y_{so}) \\
\mathbf{s.t.} \quad & X\lambda \leq \theta_l (x_o + \Delta x) \\
& Y\lambda \geq y_o + \Delta y \\
& \lambda \geq 0
\end{aligned}
$$

The other inverse DEA model is resource allocation model which is also displayed as a multi-objective programming.

$$
\begin{aligned}
(RAM) \quad \mathbf{min} \quad & (\Delta x_{1o}, \cdots, \Delta x_{mo}) \\
\mathbf{s.t.} \quad & X\lambda \leq \theta_l (x_o + \Delta x) \\
& Y\lambda \geq y_o + \Delta y \\
& \lambda \geq 0
\end{aligned}
$$

2.4 Elasticity

Elasticity is an economic concept which represents the relative change in outputs compared to the relative change in inputs. Wang et al[7] firstly define scale elasticity based on DEA literature. Elasticity analysis frequently occurs in DEA literature, but barely in inverse DEA literature[13]. Intuitively, the terminology 'elasticity' reflects the possibility and potential of scale efficiency's change, which occurs in the DEA literature as marginal production analysis. Such concept also effects in inverse DEA model. When it is extended to high dimensional case, the accurate elasticity is very complicated to get. In [16], elasticity is solved by means of MPSS (most productive scale size).

$$
\begin{aligned}
(Scale) \quad \mathbf{max} \quad & \beta/\alpha \\
\mathbf{s.t.} \quad & \beta y_o \leq \sum_{j=1}^{n} y_j \lambda_j \\
& \alpha x_o \geq \sum_{j=1}^{n} x_j \lambda_j \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \alpha, \beta \geq 0
\end{aligned} \tag{4}
$$

Suppose (α^*, β^*) is the optimal solution to model (4), then we can generalize the elasticity concept to the case of multiple-inputs and multiple-outputs by noting that:

$$
\begin{aligned}
(Elasticity) \quad \frac{1-\beta^*}{1-\alpha^*} &= \frac{y_{ro} - \widehat{y}_{ro}}{y_{ro}} / \frac{x_{io} - \widehat{x}_{io}}{x_{io}} \\
&= \frac{x_{io}}{y_{ro}} \frac{\Delta y_{ro}}{\Delta x_{io}}
\end{aligned} \tag{5}
$$

For each of these mix pairs we then have a measure of scale change associated with movements to the region of MPSS.

3 Experiment and Results

3.1 Economics Efficiency evaluation

We have stressed the urgent demand for efficiency analysis in health care, when it refers to DEA method, examples of DMU can be entire health systems, purchasing organizations, hospitals, physician practices, or individual physicians[3]. Efficiency analysis is centrally concerned with measuring the competence with which inputs are converted into valued outputs. Production frontier is built in DEA model by using the empirical data, which envelopes all the DMUs within possibility production set.

When execute DEA models for efficiency assessment in health care, a lot of details need to be considered. From the data level, first of all is the data collection:

DEA models require positive data without data absence. Then is the choice of input and output index. There is no general with which to discriminate in many of these choices and the appropriate strategy may depend on the decisions to be made and the nature of data available. Naive efficiency analysis involves examining the ratio of health system outputs to inputs. Yet system inputs should also include previous investments and exogenous inputs. And system outputs should not directly related to health, such as enhanced productivity.

From the model level, we must be circumspect when making the following opinions: whether to assume constant or variable returns to scale; whether to assume an input or output orientation; whether to apply weights restrictions; how to adjust for environment factors; how to cope with unexpected evaluation outcome (e.g. too many $\theta = 1$ DMUs come out leads to lack of discrimination for efficiency.) etc.

3.2 Bonus allocation and prediction model

As we have mentioned, post-DEA method mainly cope with problems posed after efficiency evaluation. Resource allocation problem lies in our focus as it is since the strong relationship between budget use and efficiency of health care DMUs. Suppose that the central decision maker manages several sectors, and will distribute some expenditure as bonus to all or part of sectors, how to allocate the bonus? Such budget allocation can be well solved by using extra resource allocation algorithm.

Moreover, for decision maker of health care system, two problems are mainly concerned before allocating bonus:

1) Given the efficiency result of a given sector, if certain budget is allocated to it, what is the greatest possible increment of outputs?

2) Given the efficiency result of a given sector, if manager expects a particular amount of output increment, how much budget is need at least?

Inverse DEA methods is employed to answer the above two questions. When we execute inverse DEA model, some unexpected outcomes should be retreated, among which the most important one is that the built (MOP) has no optimal solution except that we apply common weights aggregation in the objective function of (MOP). Such inappropriate situation can be eliminated by model specification. It is suggested the number of DMUs denoted by n is at least three times the number of factors $(m+s)$ in any DEA application[17]. In [16], it only asks for $n \geq 2(m+s)$.

3.3 Post-DEA process

In this paper, post-DEA method includes all the models referred to the second stage after efficiency evaluation. In practice, the most popular problem after efficiency assessment is resource allocation, where inverse DEA plays a major part. But post-DEA is not equal to inverse DEA, it also includes elasticity analysis, bonus allocation algorithms, common weights evaluation, and other models occurs in or not in DEA literature which deal with the second stage problems after DEA evaluation.

3.4 Numerical Example

10 hospitals selected from 171 hospitals in England are considered in this paper as shown in table 1. The original data is collected from [3]. Hospital 'Mean' is the mean value of 171 hospitals, which is used here as a DMU. Input index is selected as 'TOTCOST', which is the total cost or total revenue expenditure; Outputs index including: 'INPATIENTS' as total inpatient episodes weighted by HRG case mix index; 'OUTPATIENTS' as first outpatient attendances; 'A&E' as total A&E attendances.

Table 1: Data for ten hospitals.

DMU	TC	IP	OP	AE
A	113.524	0.09	0.147	0.018
B	44.258	0.033	0.042	0.073
C	66.503	0.042	0.052	0.054
D	108.679	0.092	0.105	0.057
E	110.669	0.073	0.072	0.087
F	82.237	0.049	0.081	0.061
G	37.077	0.029	0.036	0.038
H	108.908	0.087	0.102	0.079
I	67.643	0.037	0.043	0.074
J	85.284	0.05	0.053	0.076
Mean	61.553	0.043	0.053	0.058

DEA evaluation results

The following table 2 displays the efficiency scores according to different DEA models. In table 2, input oriented CCR model (CCR-I) obtains the same efficiency with the output oriented CCR model (CCR-O). But when it comes to BCC models, the efficiency score and the returns to scale results make difference from input oriented to output oriented model.

Table 2: Efficiency score of different DEA models.

DMU	CCR-I	CCR-O	BCC-I	RTS	BCC-O	RTS
A	1	1	1	C	1	C
B	1	1	1	C	1	C
C	0.794	0.794	0.798	I	0.798	D
D	1	1	1	C	1	C
E	0.817	0.817	1	D	1	D
F	0.869	0.869	0.920	D	0.949	D
G	0.986	0.986	1	I	1	I
H	0.968	0.968	1	D	1	D
I	0.722	0.722	0.741	D	0.950	D
J	0.746	0.746	0.783	D	0.931	D
Mean	0.881	0.881	0.884	I	0.882	I

Common weights evaluation results

The corresponding common weights for this example is $\theta_{cw} = (0.0072, 5.8063, 1.9681, 0.6689)$. The following table 3 provides the comparison result of efficiency scores calculated by BCC-O model and by using θ_{cw}.

Table 3: Efficiency scores of BCC-O and θ_{cw}.

DMU	BCC-O	θ_{cw}
A	1	1.008
B	1	1.014
C	0.798	0.798
D	1	0.995
E	1	0.783
F	0.949	0.819
G	1	0.991
H	1	0.968
I	0.950	0.717
J	0.931	0.725
Mean	0.882	0.886

Common weights analysis provides a benchmark for sorting DMUs whose efficiency score is equal to 1, such as hospitals $\{A, B, D, E, G, H\}$. We can not discriminate the best performance DMU from the efficient DMUs' set by means of traditional DEA models. But according to table 3, we can get the comparison result as $B > A > D > G > H > F > C > E > J > I$, which provides more information for evaluation.

Extra resource allocation

The following table 4 provides the bonus allocation weights. The exact allocation weights by using $X * \theta$ gets very close to the allocation weights by using $Y. * \theta_{cw}$, the latter one is a compromise solution compared with the former one.

Table 4: Bonus allocation weights using θ_{CCR}.

DMU	$X * \theta$	$Y. * \theta_{cw}$
A	0.154	0.157
B	0.060	0.062
C	0.072	0.073
D	0.148	0.149
E	0.123	0.119
F	0.097	0.093
G	0.050	0.051
H	0.143	0.145
I	0.066	0.067
J	0.086	0.085

Inverse DEA prediction

Suppose that the bonus total is 100, and according to allocation weights, certain bonus is allocated to each of 10 hospitals. Assume that the efficiency score is unchanged, and we employ (IPM) model to predict the maximal output increments for each hospital. The outcome is illustrated in table 5. The efficiency score of each improved DMU is also showed in the last column in table 5, which maintains the efficiency result in table 2.

Elasticity analysis

Table 5: Data for ten hospitals after bonus allocation.

DMU	TC	IP	OP	AE	θ_{CCR}
A	128.957	0.100	0.147	0.107	1
B	50.275	0.037	0.048	0.0829	1
C	73.69	0.044	0.056	0.097	0.794
D	123.453	0.092	0.117	0.204	1
E	122.962	0.0749	0.095	0.166	0.817
F	91.951	0.060	0.081	0.109	0.869
G	42.047	0.031	0.039	0.068	0.986
H	123.242	0.089	0.113	0.197	0.968
I	74.282	0.040	0.051	0.088	0.722
J	93.928	0.052	0.066	0.116	0.746

Moreover, if we take into account of returns to scale aspect, which assumes the efficiency score could be changed, then we can get a new result that more appropriate to reality[13]. Take hospital G for instance, its RTS is increasing, means that its elasticity value is greater than 1. Assumed that hospital G's scale efficiency get up to 1 when its input increment is up to 5. The expected maximal output value come out to be $(0.0314, 0.0399, 0.0694)$ which is more promising compared with the result $(0.031, 0.039, 0.068)$ in table 5.

On the other side, if we expect certain output increment of a given hospital, for example for DMU H the output increment as $\Delta y_H = (0.01, 0, 0)$, then the minimal Δx_H is 11.4069. But concerning the DMU H is of decreasing returns to scale, suppose its efficiency changes to 0.95, then the minimal Δx_H rises up to 13.4486.

4 Conclusion

This paper outlines the main issues involved in specifying DEA and post-DEA method to manage health care budget and to predict potential improvements of health care sectors. Data Envelopment Analysis(DEA) has been a commonly used tool for measuring the efficiency of health care sectors, based on which, post-DEA models are proposed for solving problems in budget management and related decision makings. The post stage faces a number of decisions models regarding common weights model for full ranking, bonus allocation, inverse DEA for prediction, and elasticity analysis for quantifying potential scale efficiency, etc. The advantages of the post-DEA framework are its freedom from parametric assumptions, and its flexibility to be combined with other framework such as game theory, statistic, and mathematical programming. The drawback of post-DEA is same with the drawback of DEA method, that they fail to offer an exact production representation and guidance on the quality of the results. However, DEA and post-DEA method, compared with other tools, can greatly meet the demands of efficiency assessment and budget management in health care systems, which has great potential in future applications.

Acknowledgments

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