ENUMERATION OF REGION PARTITIONING FOR EVACUATION PLANNING BASED ON ZDD

Atsushi Takizawa[1], Yasufumi Takechi[2], Akio Ohta[2], Naoki Katoh[2], Takeru Inoue[3], Takashi Horiyama[4], Jun Kawahara[5], Shin-ichi Minato[3, 6]

[1]*Department of Architecture and Building Engineering, Osaka City University, Japan*

[2]*Department of Architecture and Architectural Engineering, Kyoto University, Japan*

[3]*Minato Discrete Structure Manipulation System Project, ERATO, Japan Science and Technology Agency, Sapporo, Japan*

[4]*Graduate School of Science and Engineering, Saitama University, Japan*

[5]*Graduate School of Information Science, Nara Institute of Science and Technology, Japan*

[6]*Graduate School of Information Science and Technology, Hokkaido University, Sapporo, Japan*

Keywords: evacuation planning, region partitioning, enumeration, zero-suppressed BDD, reverse search.

Abstract

Japanese cities always have risks of large-scale earthquakes. Thus, it is very important to establish crisis management systems against large-scale disasters such as big earthquakes, and tsunamis to secure evacuation centers for evacuees. In this respect, it is extremely important to provide sufficient evacuation centers and to appropriately partition the whole region into small areas, such that a unique evacuation center is located in each area and the people living in the area can easily evacuate to the center. However, it is hard to find an optimal region partitioning, due to the uncertainty, such as a fire or collapsed buildings. In this research, we propose a method to enumerate all partitioning patterns using Zero-suppressed Binary Decision Diagram (ZDD) that satisfy several conditions. We apply the proposed method to Kamigyo Ward of Kyoto City, Japan.

1 Introduction

Japanese cities always have risks of large-scale earthquakes. **Figure 1** shows the estimated occurrence probability of a large-scale earthquake in Japan from 2012 to 2042, which was published by Japanese Headquarters for Earthquake Research Promotion [6].

The occurrence probability of a large-scale earthquake in Kyoto City, which is one of the largest cities in western Japan, is estimated as 13.6% in this report, and this is relatively high. It is considered that the earthquake risk of Kyoto is mainly due to Hanaore Fault. This earthquake is considered to cause many uncertain accidents like fires or building collapses, because Kyoto has many narrow streets and congested timeworn wooden houses. Especially many of them are located in Kamigyo Ward of Kyoto City, which is a target area in our research (**Figure 2**). The Kyoto City office’s report estimates that there may have about 20000 evacuees from this earthquake [4]. If this earthquake occurs, facilities such as school gymnasiums will be used as evacuation centers for the evacuees to stay. Although some local governments designate evacuation centers to every area, Kyoto City does not. People seem to go to the nearest center if there are no designations, and this may make the situation more chaotic because the capacity of evacuation centers is normally lower than the number of evacuees in a large-scale disaster.

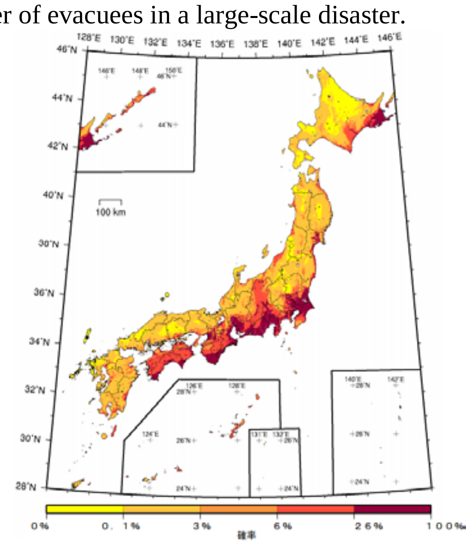

Figure 1. The estimation of occurrence probability of earthquakes with a seismic intensity of at least lower 6 on the seven-point Japanese scale from 2012 to 2042 [6].

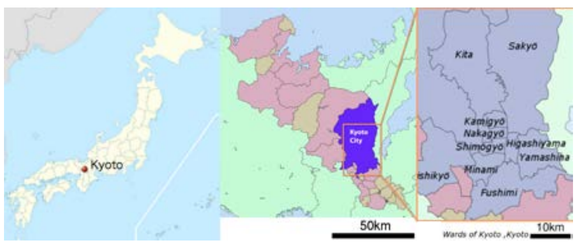

Figure 2. The location of Kamigyo Ward. Figures are from Wikipedia/Kyoto [10].

Thus, we need to appropriately partition the whole region into small areas in each of which exactly one evacuation center is located and the evacuees in the area are designated to go to that evacuation center. The appropriateness of region partitioning should be measured by several criteria. Thus, it is hard to find the best partition. Instead, it is better to enumerate all possible partitioning patterns among which people can compare the patterns from various viewpoints. However, the number of possible partitioning patterns is enormous. Therefore, for this purpose we use Zero-suppressed Binary Decision Diagram (ZDD) (Minato, 1993), which is a data structure that can represent a family of sets compactly.

Takematsu (2006) suggested that there may exist partitioning patterns more balanced than that in which people use their nearest evacuation center. However, this only suggests that better patterns exist, and did not mention how to find them. The one possible way to do this is to first enumerate all possible patterns and then to extract better patterns. However, it is impossible to enumerate all possible partitioning patterns mainly because of the computational and space complexity, but we enable it by using ZDD.

ZDD was first introduced by Minato (1993). Currently there are many applications of ZDD in various fields and to various problems. For instance, Kawahara et al. (2011) introduced a method to enumerate paths of a graph and Inoue et al. (2012) also shows the application to smart grid, in which all possible electrical distribution network patterns are enumerated.

This paper is organized as follows. We briefly explain ZDD in Section 2. Then in Section 3, we show the overview of the enumeration algorithm of partitioning patterns satisfying several constraints. Section 4 presents an enumeration algorithm for finding all feasible regions that contain a particular evacuation center. In Section 5, based on the algorithm in Section 4 we present a method for enumerating all feasible region partitioning patterns. In Section 6, we apply this method to Kamigyo Ward of Kyoto City. We conclude this paper in Section 7.

2 ZDD (Zero-suppressed Binary Decision Diagram)

ZDD, a Zero-suppressed Binary Decision Diagram, is first introduced by Minato (1993) (see also Knuth (2009)), which can represent a family of sets more compactly than BDD, a binary decision diagram (Akers, 1978). ZDD requires the following two additional reduction rules to BDD.

(i) Eliminate the nodes whose 1-edge points to the 0-terminal node. Then connect the edge to the other subgraph directly.

(ii) Share all equivalent subgraphs.

Figure 3 illustrates an example of BDD and ZDD for sets $\{e_2, e_3, e_4\}$ and $\{e_1, e_4\}$ on the ground set $\{e_1, e_2, e_3, e_4\}$. For more detail, see Minato (1993).

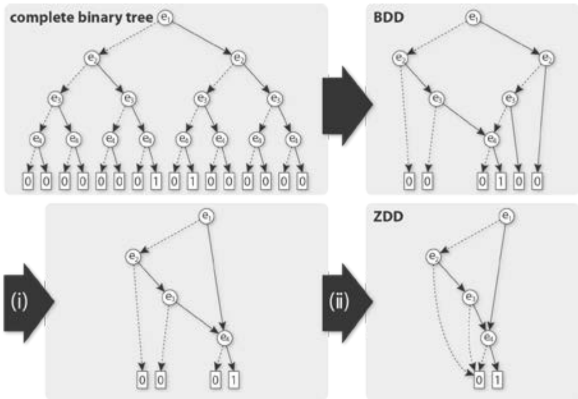

Figure 3. An example of BDD and ZDD reductions from a complete binary tree for sets $\{e_2, e_3, e_4\}$ and $\{e_1, e_4\}$ on the ground set $\{e_1, e_2, e_3, e_4\}$.

There are continued active discussion and implementation of family of set operations for ZDD in *JST ERATO Minato Discrete Structure Manipulation System Project [7]*.

3 Overview of the enumeration algorithm of region partitioning

3.1 Target district

As mentioned in Section 1, we consider Kamigyo Ward of Kyoto City as target district in this research. This district is divided into complex shaped administrative areas. It is ideal to take this area as a unit of the partitioning, but we cannot because of the following two reasons. One is that the divided areas are not balanced in their size, and the other reason is that the areas often have too complicated shapes to construct models with which we can execute the algorithm with reasonable amount of time. Therefore, we adopt a square cell as the unit of partitioning and divide the target district into the cells. Note that we need to determine the size of the cell on a trade-off between computation time vs. accuracy. **Figure 4** illustrates a partition of the target district of Kamigyo Ward into cells. We set the cell to 200m by 200m square, and the number of the cells is 138. We consider only the west of the Karasuma Street because there is Kyoto Imperial Palace in the east of Kamigyo Ward where nobody lives.

In this research we consider a short time refuge which means the place where people are forced to live in the evacuation center by a disaster, but does not mean the one to gather for securing safety or wait for recovery of transportations systems in an emergency. Actually, facilities like school gymnasiums are often designated as evacuation centers. In **Figure 4**, a red cell indicates that an evacuation center exists therein. There are 17 evacuation centers in the target district and the sum of their capacity is 5,296 people. Note that the population of the target district is 73,678 in 2012.

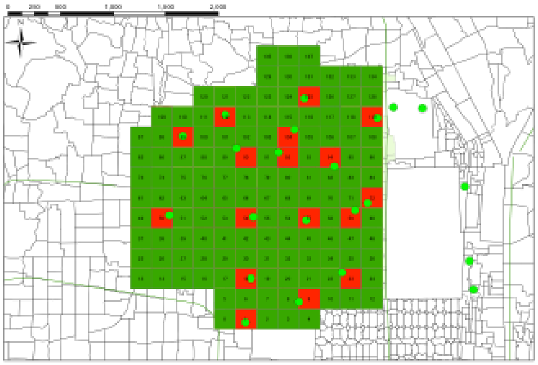

Figure 4. An illustration of the partition of Kamigyo Ward into cells. A red cell indicates that an evacuation center exists in the area.

3.2 Overview of the proposed method

The proposed method consists of two steps. Let the set of evacuation centers be denoted by $\{c_1, c_2, ..., c_{17}\}$. For each evacuation center c_1, the first step enumerates all feasible regions that contain only c_1, and satisfy three constraints: convexity, distance and capacity constraints that will be explained in the next subsection. Let $\mathfrak{R}_1$ be the set of regions obtained for evacuation center $c_i, i = 1, ..., 17$. $\mathfrak{R}_1$ is then converted into ZDD. The second step aggregates $\mathfrak{R}_i, i = 1, ..., 17$ and computes a collection of all possible partitioning patterns $\wp$ such that every $P \subset \wp$ partitions the target district D into $R_1, ..., R_{17}$ i.e.,

$$R_1 \subset \mathfrak{R}_1 (i = 1, ..., 17), \bigcup_{i=1}^{17} R_i = D, R_i \cap R_j = \varnothing (i \neq j). \quad (1)$$

The overall framework of the proposed method is shown in **Figure 5**.

Figure 5. The overall framework of the proposed method.

3.3 Constraints

We consider the following three constraints to define feasible regions that contain an evacuation center.

Convexity constraint

Let us consider an evacuation center $\hat{c}$. A region R that contains $\hat{c}$ is called *convex* if it is expressed as a union of rectangles each of which contains $\hat{c}$ (See **Figure 6**). The concept of the convex region used in this paper was first introduced by Chen et al. (2004).

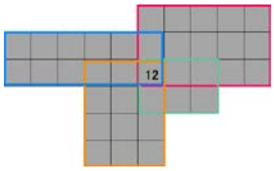

Figure 6. An example of region R which satisfies the convexity constraint. An evacuation center is located in the cell 12.

If an R satisfies *the convexity constraint*, there are at least two edge-disjoint rectilinear shortest paths from each cell in R to $\hat{c}$ except for the cells on the vertical or horizontal straight line passing through $\hat{c}$ (see **Figure 7**) where the rectilinear shortest path is the unique straight line.

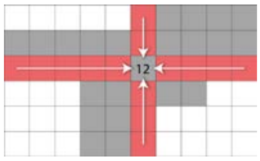

Figure 7. In the convex region R which is the same as Figure 6, each red cell has the unique rectilinear shortest path to cell 12.

Figure 8 shows an example that does not satisfy *the convexity constraint*.

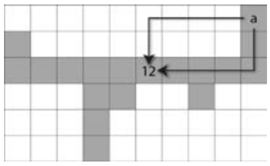

Figure 8. An example of non-convex region.

We consider a region R that satisfies *the convexity constraint* with respect to cell $\hat{c}$. Let c_{top} be the upper most cell in R which is on the vertical line passing through $\hat{c}$. Similarly c_{bottom}, c_{left} and c_{right} are defined. A cell c is *on the x-axis* (resp. *y-axis*) with respect to $\hat{c}$ if the rectilinear shortest path from c to $\hat{c}$ is a straight vertical or horizontal line. See **Figure 9**. **Figure 9** (a) illustrates cells contained in R as gray cells. **Figure 9** (b) illustrates the cells which are not in R that are adjacent to R as yellow cells. **Figure 9** (c) illustrates cells $c' \notin R$ such that $R \cup c'$ is convex, which are colored in red or blue. We call these cells *eligible*. A cell $c' \notin R$ is *eligible* if c' is adjacent to $c \in R$ and either

(i) c' is *on the x-axis* or *y-axis* with respect to $\hat{c}$ (red cells), or

(ii) c' is on the concave corner of R (blue cells).

Cells on the concave corner of R are those adjacent to two of boundary cells of R. We obtain the following lemma (the proof is omitted).

Lemma1: Suppose R is a convex region with respect to the center $\hat{c}$. $R \cup c$ is convex if and only if c is *eligible* with respect to R.

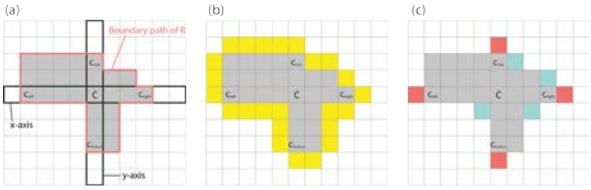

Figure 9. An example of eligible cells with respect to region R. A set of gray cells stands for R. Red and blue cells are eligible.

Distance constraint

Distance to the evacuation center should not be so far. We set the upper bound of distance to 1,100m.

Capacity constraint

In case of Kyoto City, the capacity of evacuation centers is less than half of the expected number of evacuees. This means we may have to accept evacuees over capacity. Even if so, we need to consider a ratio of number of evacuees to the capacity. Practically, the ratio should be chosen as low as possible to seek for well-balanced solutions. Additionally, we assume that the number of evacuees is proportion to the population in a cell in the case of the earthquake caused by Hanaore Fault, which we consider, because almost all areas in Kamigyo Ward are estimated to have the same level of quake. However, the regional distribution of the predicted evacuees cannot be estimated.

3.4 Evaluation criteria for partitioning patterns

After extracting all possible region patterns satisfying three constraints explained in Section 3.3, we need criteria to evaluate the appropriateness of region partitioning.

Criterion 1. The uniformity of the ratio of assigned evacuees to the capacity of evacuation center.
Criterion 2. The sum of distances from cells to assigned evacuation centers.

The uniformity of ratio is measured by the standard deviation of ratios. Note that we assume the evacuation completion time is proportional to the distance from the cell to the evacuation center.

4 Enumeration of all region patterns for a particular evacuation center

In this section, we explain a method to enumerate all feasible regions for a particular evacuation center using *reverse search.*

4.1 Reverse search

Avis and Fukuda (1996) introduced reverse search that is a memory efficient method for visiting all the nodes of a connected graph that can be defined implicitly by an adjacency oracle. It can be used whenever a spanning tree of the graph can be defined implicitly by a parent function. This function is defined for each vertex of the graph except a prespecified root. Iterating the parent function leads to a path to the root from any other vertex in the graph. The set of such paths defines a spanning tree, known as the search tree. The method is used to various enumeration problems such that for a finite set S and F which is a family of subsets of S and in which an adjacency relationship between elements of F is implicitly given, we are asked to enumerate all elements of F.

4.2 Preparation

Let us consider a square area satisfying *the distance constraint* whose center cell is an evacuation center. As an example, suppose *the distance constraint* is 300m. Let us consider 3×3 square area as target for simplicity. Cells are indexed as in **Figure 10**.

6	7	8
3	4	5
0	1	2

Figure 10. 3×3 square area and indices of cells.

Let G denote the set of cells in this square area, and denote the cell of G with index i as c_i. Denote the estimated number of evacuees at a cell by function $s: G \mapsto Z_+$, where Z_+ is the set of natural numbers.

As we explained in the previous subsection, *reverse search* is a method for visiting all the nodes of a connected graph. In this research, the node of the graph is a feasible region R that contains the target evacuation center cell $\hat{c}$. Then we will define the root, the adjacency function, and the parent function required by *reverse search*.

The root R_{root}

Define a region that contains only the target evacuation center cell as the root. Namely, $R_{root} = \{\hat{c}\}$.

The adjacency function Adj

Let $\delta(R)$ denote the number of the *eligible* cells with respect to a convex region R. And the adjacency function $Adj(R,j) j < \delta(R)$ returns a region $R \cap c$ where c is a cell which has the jth minimum index in the *eligible* cells with respect to R. If there are no *eligible* cells, the adjacency function returns null.

The parent function f_{parent}

Reverse search requires the existence of the parent to every node of graphs except the root. Suppose R is a convex region with respect to the center $\hat{c}$, and let R'_c be a region which is obtained by removing a cell $c \in R$. Let $c_{R_{max}}$ denote the cell which has the maximum index in cells that are contained in R and R'_c is convex. We define $R'_{c_{R_{max}}}$ as the parent of R. Therefore the parent function $f_{parent}(R)$ returns $R'_{c_{R_{max}}}$.

4.3 Enumeration Algorithm

The following algorithm enumerates all feasible regions with respect to the target evacuation center $\hat{c}$.

The number of evacuees in R is $\sum_{c \in R} s(c)$ and by using this we can check whether a region R satisfies *the capacity constraint* or not.

Algorithm 1: Enumerate all regions for an evacuation center

1. $R \leftarrow R_{root}$; $j \leftarrow 0$;
2. **repeat**
3. **while** $j < \delta(R)$ **do**
4. $j \leftarrow j+1$;
5. **if** $Adj(R, j) \neq null$ **and** $f_{parent}(Adj(R, j)) = R$ **then**
6. $R' \leftarrow Adj(R, j)$;
7. **if** R' does not satisfy *the capacity constraint* **or** *the distance constraint* **then**
8. **go to** line 4;
9. **end if**
10. $R \leftarrow R'$; $j \leftarrow 0$;
11. $Output(R)$;
12. **end if**
13. **end while**
14. **if** $R \neq R_{root}$ **then**
15. $R'' \leftarrow R$; $R \leftarrow f_{parent}(R)$;
16. $j \leftarrow 0$;
17. **repeat** $j \leftarrow j+1$ **until** $Adj(R, j) = R'$
18. **end if**
19. **until** $R = R_{root}$ **and** $j = \delta(R_{root})$

Let us analyze the time complexity required per output. For this let us consider the region R considered in the algorithm. The adjacency function is computed in $O(1)$ because the *eligible* cells are determined by those of the parent and the neighbor of a cell $c \in R, c \notin f_{parent}(R)$. In function f_{parent} we check whether the convexity constraint is satisfied or not, and it is also computed in $O(1)$. After that, we check the feasibility of the capacity constraint and the distance constraint. Although both of these require the sum of function for each cell in R, it can be computed in $O(1)$ by using the result of the parent. Thus, the time spent by outputting one feasible region $O(n)$. **Figure 11** illustrates the example of all feasible regions for 3×3 square area that are enumerated by *Algorithm 1*.

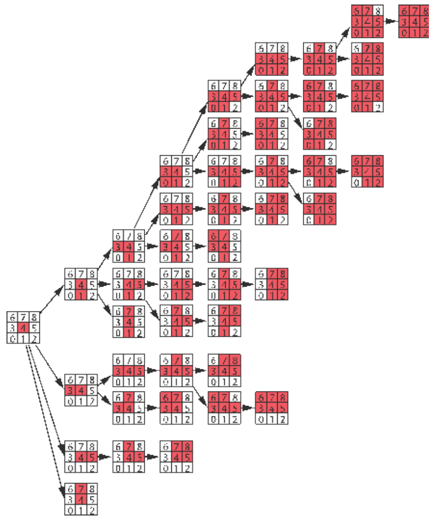

Figure 11. An example of feasible regions in a 3×3 square area. Cell 4 represents an evacuation center.

5 Integrating the set of regions for all evacuation centers

We explain the algorithm that enumerates all partition patterns by using ZDD (see algorithm 2). In algorithm 2, ZDD represents the one that eventually becomes the solution to be output and ZDD_i represents partition patterns that correspond to $\mathfrak{R}_i$ that was obtained by Algorithm 1. We initialize ZDD as ZDD_1. Then in each iteration $i \geq 2$, we update ZDD as the Cartesian product of ZDD and ZDD_i. Each updated ZDD may not satisfy (1). Such ZDDs contain either cells that are not assigned to any evacuation center to use or multiple evacuation centers. To remove those patterns, we use $Restrict(zdd)$ method in the software developed by *JST ERATO Minato Discrete Structure Manipulation System Project* and Minato (2001). This method makes new ZDD such that every vector represented by the ZDD contains at least one of combinations represented by zdd (argument). For this we construct two ZDDs, $OZDD$ and $SZDD$. $OZDD$ represents all patterns that contain cells that are assigned to more than one evacuation center, and $SZDD$ represents all patterns such that all cells are assigned to at least one of the evacuation centers.

Algorithm 2: Enumerate all feasible partition patterns of target district.

1. $ZDD \leftarrow \varnothing$, construct $OZDD$ and $SZDD$
2. **for** $i \leftarrow 1$ **to** 17
3. Apply Algorithm 1 for evacuation center c_i and obtain the set of regions $\mathfrak{R}_i$
4. Construct ZDD_i for $\mathfrak{R}_i$
5. **if** i **is** 1
6. $ZDD \leftarrow ZDD_i$
7. **else**
8. $ZDD \leftarrow ZDD \times ZDD_i$

$ZDD \leftarrow ZDD \setminus ZDD.\mathrm{Restrict}(OZDD)$
$ZDD \leftarrow ZDD.\mathrm{Restrict}(SZDD)$
end if
end for

In implementing Algorithm 2, ZDD is represented as in **Figure 12**. More precisely first index all cells of target district D as of **Figure 4**. There are 138 cells and 17 evacuation centers in the target district that we consider, therefore a region partitioning pattern $P \in \wp$ can be represented by a 17×138 *dimensions* $0-1$ *vector*. Hence set of all partitioning patterns $\wp$ is represented by ZDD as in **Figure 12**. In **Figure 12**, the node of index n corresponds to a cell of index $i = \lfloor n/17 \rfloor$ where $\lfloor x \rfloor$ denotes the integer part of x.

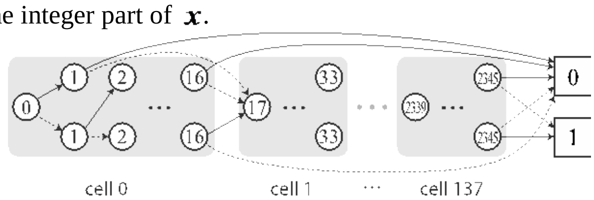

Figure 12. Illustration of ZDD that is constructed by Algorithm 2.

6 Application to Kamigyo Ward, Kyoto City

We applied this method to the district in Kamigyo Ward, which was shown in **Figure 4**. We tried three different sets of conditions for the enumeration. These conditions and the results are as follows.

Condition 1.

- *the distance constraint*: 1,100m
- *the capacity constraint*: 1,500%
- the number of patterns output: 0

Condition 2.

- *the distance constraint*: 1,100m
- *the capacity constraint*: 1,600%
- the number of patterns output: 7,126,383

Condition 3.

- *the distance constraint*: 1,100m
- *the capacity constraint*: 1,545%
- the number of patterns output: 91,520

The case the value of *the capacity constraint* is 1,500% (*Condition 1*) did not produce any feasible pattern, and thus next we raised the value to 1,600% and then we obtained solutions, but too many to evaluate (*Condition 2*). Finaly we obtained 91,520 patterns for *Condition 3*. We then evaluate the set of solutions obtained under condition 3.

We consider two criteria shown in Section 3.4 for measuring the goodness of region partitioning of evacuation centers.

Criterion 1 is measured as the standard deviation of capacity ratios. *Criterion 2* is measured as the sum of distances from cells to the assigned evacuation center. Of course for both of these, the lower the value becomes, the better solution we obtain.

We plotted the values of those two criteria of the 91,520 patterns in **Figure 13**. It is reasonable that a decision maker should choose the one from among a set of solutions on the efficient frontier.

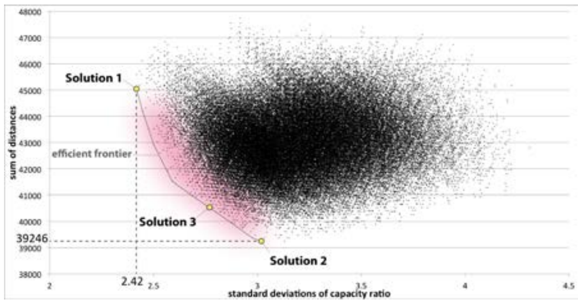

Figure 13. Illustration of the values for two criteria for 91,520 partition patterns.

We show some partitioning patterns. Figure 14 illustrates the population in each cell by gray scale. Evacuation centers are located in numbered cells. Table 1 shows the capacity of each evacuation center.

The optimal partition pattern with respect to *Criterion 1* is illustrated in Figure 15, and its objective values of Criteria 1 and 2 are 2.42 and 45069, respectively, which is plotted as solution 1 in Figure 13.

The optimal partition pattern with respect to *Criterion 2* is illustrated in Figure 16, and its objective values of Criteria 1 and 2 are 3.03 and 39246, respectively, which is plotted as solution 2 in Figure 13.

One of the pareto optimal partition patterns is illustrated in Figure 17, and its objective values of Criteria 1 and 2 are 2.77 and 40540, respectively, which is plotted as solution 3 in Figure 13.

We also computed the region partition that optimizes *Criterion 2* without taking into consideration three constraints (convexity constraint, and capacity constraint, distance constraint). This solution may be proposed by a practitioner. However, the objective value of Criterion 1 may become large. In fact, objective values of Criteria 1 and 2 are 9.87 and 34030, respectively. Thus, such solution may not be acceptable. (See Figure 18.)

id	capacity	id	capacity
1	320	10	432
2	212	11	473
3	396	12	131
4	280	13	375
5	352	14	187
6	288	15	288
7	204	16	384
8	356	17	298
9	320		

Table 1: Capacity of evacuation centers.

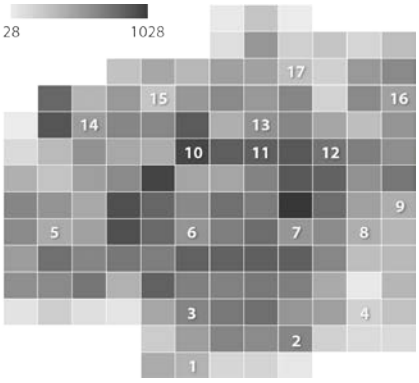

Figure 14. Number of population in each cell.

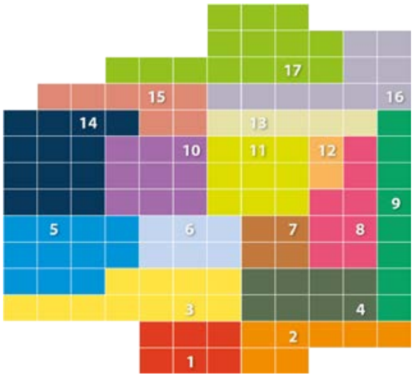

Figure 15. The optimal pattern with respect to Criterion 1 (Solution 1).

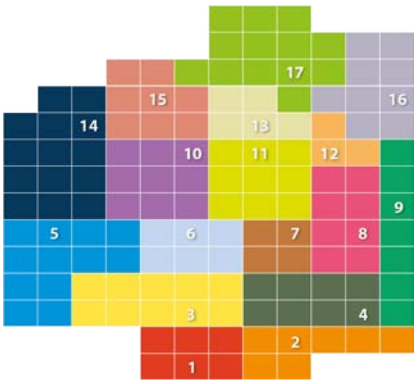

Figure 16. The optimal pattern with respect to Criterion 2 (Solution 2).

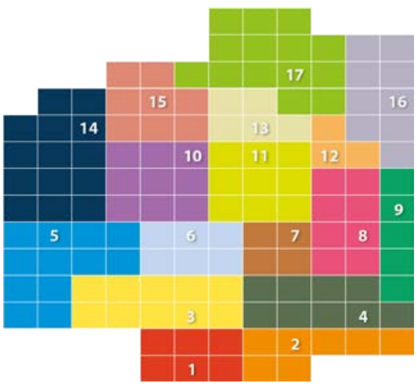

Figure 17. One of the pareto optimal patterns (Solution 3).

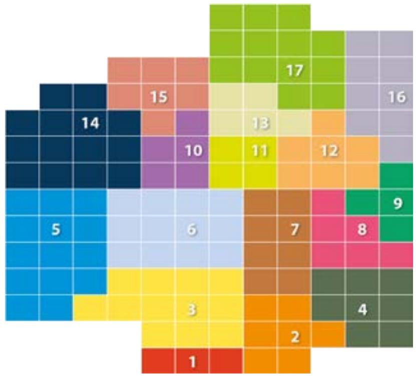

Figure 18. The optimal pattern with respect to Criterion 2 without considering three constraints.

7 Conclusion

The paper studied the problem of region partitioning into subregions which arises in evacuation planning. Given a set of evacuation centers in a given regions we are asked to find an appropriate partition of the region into subregions in such a way that a subregion contains exactly one evacuation center subject to several constrains; the shape of a subregion, the maximum distance from a cell to evacuation center, and the ratio of the population in the subregion to the capacity of the evacuation center.

Since we need to consider criteria to measure the appropriateness of the partitioning, we first enumerate all feasible partitioning patterns, and evaluate them according to the criteria, the proposed method is based on the reverse search and ZDD, and was applied to the Kamigyo ward of Kyoto City to demonstrate the effectiveness of the method.

References

[1] S. B. Akers: "Binary Decision Diagrams", IEEE Trans. Comput., pp. 509-516, 1978.

[2] D. Avis, K. Fukuda, "Reverse Search for Enumeration", Discrete Applied Math, Vol. 6, pp. 21-46, 1996.

[3] D. Chen, J. Chun, N. Katoh, T. Tokuyama, "Efficient Algorithms for Approximating a Multidimensional Voxel Terrain by a Unimodal Terrain", In: Proc. 10th Ann. Internat. Conf. Computing and Combinatorics (COCOON'04). Lecture Notes in Comput. Sci., vol. 3106, pp. 238–248. Springer, Berlin, 2004.

[4] City of Kyoto, http://www.city.kyoto.lg.jp/gyozai/cmsfiles/contents/0000086/86399/02EA_kamigyo_map_jishin.pdf, accessed May 28, 2013.

[5] T. Inoue, K. Takano, T. Watanabe, J. Kawahara, R. Yoshinaka, A. Kishimoto, K. Tsuda, S. Minato, and Y. Hayashi, "Loss Minimization of Power Distribution Networks with Guaranteed Error Bound", Technical report, Hokkaido University, Division of Computer Science, TCS Technical Reports, TCS-TR-A-12-59, 2012.

[6] Japanese Headquarters for Earthquake Research Promotion, "Study on assessment of earthquake hazard of future", (in Japanese), http://www.jishin.go.jp/main/chousa/12_yosokuchizu/index.htm, accessed May 28, 2013.
[7] "*JST ERATO Minato Discrete Structure Manipulation System Project* ", http://www-erato.ist.hokudai.ac.jp/, accessed May 28, 2013.
[8] J. Kawahara, T. Saitoh, H. Suzuki, S. Minato, R. Yoshinaka, "Enumeration Paths Using ZDD", (in Japanese). RIMS Kôkyûroku No.1744, pp.35-41, 2011.
[9] D. Knuth, "The Art of Computer Programming, Volume 4, Fascicle 1: Bitwise Tricks & Techniques; Binary Decision Diagrams", Addison-Wesley Professional, ISBN-10: 0321580508, 2009.
[10] "Kyoto - Wikipedia, the free encyclopedia", http://en.wikipedia.org/wiki/Kyoto, accessed May 28, 2013.
[11] S. Minato, "Zero-suppressed BDDs and their applications," Int. J. Softw. Tools Technol. Transf. (STTT), vol.3, no.2, pp.156–170, Springer 2001.
[12] S. Minato, "Zero-Suppressed BDDs for set manipulation in combinatorial problems", In Proc. of the Design Automation Conference, pp.272-277, 1993.
[13] H. Takematsu, "Assignment of evacuation centers considering a fire and collapsed buildings", (in Japanese). The Operations Research Society of Japan Autumn workshop Abstracts 2006, pp.16-17, 2006.