JOINT PRICING AND PRODUCTION DECISIONS FOR NEW AND REMANUFACTURED PRODUCTS

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Abstract

This paper studies a two-period model in which a monopolistic manufacturer needs to decide the quantity of new products in the first period, as well as the selling price and quantity of remanufactured products in the second period, in order to maximize his total profit. The market size is fixed and the market segmentation is realized by the consumer’s net utility. We focus on the competition between new and remanufactured products, and provide the manufacturer’s optimal pricing and production strategies with respect to different parameter settings.

1 Introduction

It has been widely recognized that a manufacturer may create values from remanufacturing process. Instances from industry such as Kodak, IBM, and Xerox, have demonstrated the existence of a big market for remanufactured products (see Berko-Boateng et al., 1993; Toktay et al., 2000, etc). Generally speaking, remanufacturing can be profitable in saving energy consumption, raw materials and places especially when environmental issues have become more and more socially concerned. In addition, it has even been treated as a marketing strategy in Atasu et al. (2008) aside from the economic analysis of its potential profitability.

Although remanufacturing can be profitable, the decisions on joint pricing and production for the remanufactured products appear quite difficult because the remanufactured products can obviously affect the sales of primary products. The manager may have no idea about how great it is and fear the cannibalization from the remanufactured products. Therefore, it is essential for the manufacturer to make effective remanufacturing decisions.

This paper considers a two-period model in which a monopolistic manufacturer can offer both new and remanufactured products. The market size is fixed and the market segmentation is realized by the consumer’s net utility. In the first period, the manufacturer can only sell new products. In the second period, the manufacturer can recycle and remanufacture the used products that are sold in the first period, and sell them back into the market with a lower selling price. Meanwhile, the manufacturer can still sell new products, but with a higher variable cost. The number of used products available for remanufacturing in the second period is handled by an exogenous return rate. Prior to the selling season, the manufacturer has to decide the quantity of new products in the first period, as well as the selling price and quantity of remanufactured products in the second period, in order to maximize his total profit.

In literature, some papers have addressed the pricing issues on new and remanufactured products. For example, Majumder and Groenevelt (2001) as well as Ferrer and Swaminathan (2006) assume that consumers do not discriminate new and remanufactured products offered by an OEM, while value less the remanufactured products offered by an entrant. The authors examine the pricing and remanufacturing decisions of the OEM when facing competition from a local re-manufacturer, and discuss how the third party remanufacturing can induce the competition behavior when the remanufactured products cannibalize the demand for the original products. Savaskan et al. (2004) consider the choice of an appropriate reverse channel for a collection of used products from consumers, study the coordination of channels, and discuss the pricing and choice of return rate among three different channels. Debo et al. (2005) investigate the joint technology selection and pricing decisions for new and remanufactured products, and extend the results to competing re-manufacturers. Savaskan and van Wassenhove (2006) discuss the impact of remanufacturing on competing retail stores. Atasu et al. (2008) bring a marketing perspective to remanufacturing through a focus on demand factors.

Our paper differs from the existing literature in that (a) we study the joint pricing and production decisions for new and remanufactured products; and (b) in the second period, the number of used products available for remanufacturing is bounded by the quantity of new products that are sold in the first period.

The rest of this paper is organized as follows: the next section sets up a two-period model and gives de-
tailed analysis, the third section derives the optimal solutions with respect to various parameter settings, and the final section concludes this paper.

2 Methods

2.1 Model

Consider a two period model where a monopolistic manufacturer can only sell new products with an exogenous unit price $p_1$ at a unit cost $c_1$ in the first period, but can sell both new products with the unit price $p_1$ at a unit cost $c_2$ as well as remanufactured products with a lower unit price $p_2$ ($< p_1$) at a unit cost $h$ in the second period. Before the selling season, the manufacturer has to decide the supply quantity, $x$, of new products in the first period, the unit price, $p_2$, and quantity, $y$, of remanufactured products in the second period, in order to maximize his total profit.

Assume consumers are heterogeneous and the consumer’s valuation of a new product $v$ is uniformly distributed on $[0, \bar{R}]$, while a consumer’s valuation of a remanufactured product is $\theta v$, $(0 < \theta < 1)$, where $\theta$ is a depreciating factor.

The market segmentations of purchasing a new product, a remanufactured product and nothing can be realized by the consumer’s net utility with a fixed market size $A$. The net utilities for a consumer to purchase a new product and remanufactured product are $u_1 = v - p_1$ and $u_2 = \theta v - p_2$, respectively. Then, a consumer will purchase a new product if and only if $u_1 \geq u_2$ and $u_1 \geq 0$, and will purchase a remanufactured product only if $u_2 \geq u_1$ and $u_2 \geq 0$. Consequently, the probabilities for a consumer to purchase a new ($\alpha$) or remanufactured ($\beta$) product are

$$\alpha = p \left( v > \frac{p_1 - p_2}{1 - \theta} \right) = \Phi \left( \frac{p_1 - p_2}{1 - \theta} \right)$$

$$\beta = \Phi \left( \frac{p_2}{\theta} \right) - \Phi \left( \frac{p_1 - p_2}{1 - \theta} \right)$$

With the assumption of uniform distribution, we have

$$\alpha = 1 - \frac{p_1 - p_2}{R(1 - \theta)}$$

$$\beta = \Phi \left( \frac{p_2}{\theta} \right) - \Phi \left( \frac{p_1 - p_2}{1 - \theta} \right) = \frac{p_1 \theta - p_2}{R(1 - \theta) \theta}$$

Note that in (1), $\beta$ is only meaningful when $p_1 \theta - p_2 \geq 0$. When $p_1 \theta - p_2 < 0$, let $\beta = 0$, which implies that the price $p_2$ of remanufactured product is set so high that a consumer always prefers to purchase a new product. Namely,

$$\beta = \begin{cases} 
\frac{p_1 \theta - p_2}{R(1 - \theta) \theta} & \text{if } p_1 \theta > p_2 \\
0 & \text{if } p_1 \theta \leq p_2 
\end{cases}$$

Since $\alpha$ and $\beta$ depend on $p_1$ and $p_2$, there may exist a cannibalization in the second period. The market shares corresponding to new ($A \alpha$) and remanufactured products ($A \beta$) will be deterministic once $p_2$ is decided.

Assume the return rate of the used new products sold in the first period, $\gamma$, $(0 \leq \gamma \leq 1)$, is exogenous. Obviously, the manufacturer cannot prepare more than $\gamma x$ remanufactured products, and thus $0 \leq y \leq \gamma x$ hold. The price and costs should satisfy $h < c_1 < c_2 < p_1$ and $h < p_2$. It is reasonable to assume that $c_2 > c_1$ because the cost of production during the selling season is usually greater. Besides, the condition $c_1 \theta \geq h$ should hold because with $p_2 < \theta p_1$, $h > \theta c_1$ indicates that a remanufactured product will have a higher cost but a lower selling price in comparison with a new product.

The manufacturer’s objective is

$$\text{max} \quad \Pi_{12} = \max \{(p_1 - c_1)x + (p_2 - h)y \} \quad (2)$$

s.t. $0 \leq x \leq A \alpha$, $0 \leq y \leq \min\{\gamma x, A \beta\}$

2.2 Analysis

By substituting $\alpha$ and $\beta$ in (1) into (2), we have

$$\text{max} \quad \Pi_{12} \left\{ \frac{(p_1 - c_1)x + (p_2 - h)y}{x, p_2, y} \right\}$$

$$+ (p_1 - c_2) \left\{ A \left[ 1 - \frac{p_1 - p_2}{R(1 - \theta)} \right] - x \right\} \quad (3)$$

s.t. $0 \leq x \leq A \left[ 1 - \frac{p_1 - p_2}{R(1 - \theta)} \right]$

$$0 \leq y \leq A \frac{p_1 \theta - p_2}{R(1 - \theta) \theta}$

$$0 \leq y \leq \gamma x$$

The feasible region, $F$, in terms of $x$ and $p_2$ consists of $h < p_2 < \theta p_1$ and $0 \leq x \leq A \alpha$, as shown in the shaded area in Figure 1, where $x \leq A \alpha$ can be readily transformed into a linear relationship between $x$ and $p_2$ with the first constraint in (3).

The manufacturer’s profit function $\Pi_{12}$ is continuous and differentiable on the feasible region $F$, and thus its optimal solution lies on the extreme points in $F$ or on the boundaries of $F$. However, the boundary solution cannot exist on $x = 0$, i.e., on line $a_1 a_2$ in Figure 1, for which (3) will always have a better solution when $x = A \alpha$. Similarly, the boundary solution cannot exist on $p_2 = h$ or $p_2 = \theta p_1$, i.e., on line $a_1 a_4$ or line $a_2 a_3$ in Figure 1, for which (3) will always have a better solution when $h < p_2 < \theta p_1$.

Besides, the boundary solution cannot exist on $y = 0$ either, for which (3) will always have a better solution when $y > 0$. Consequently, the boundary solution can only exist on $x = A \alpha$, $y = \gamma x$ and $y = A \beta$. In what follows, we will use the Lagrangian multiplier method to solve (3).
Figure 1: Feasible Region

The Lagrange corresponding to (3) is

\[
L(x, p_2, y) = (p_1 - c_1)x + (p_2 - h)y + (p_1 - c_2) \left\{ A \left[ 1 - \frac{p_1 - p_2}{R(1 - \theta)} \right] - x \right\} + \lambda_1 \left\{ A \left[ 1 - \frac{p_1 - p_2}{R(1 - \theta)} \right] - x \right\} + \lambda_2 (\gamma x - y) + \lambda_3 \left\{ \frac{A(p_1 \theta - p_2)}{R(1 - \theta) \theta} - y \right\}
\]

With the first order optimality conditions, we have

\[
\begin{align*}
\frac{\partial L}{\partial x} &= p_1 - c_1 - p_1 + c_2 - \lambda_1 + \lambda_2 \gamma = 0 \\
\frac{\partial L}{\partial p_2} &= A(p_1 - c_2) + y + \frac{A\lambda_1}{R(1 - \theta)} - \frac{A\lambda_3}{R(1 - \theta) \theta} = 0 \\
\frac{\partial L}{\partial y} &= p_2 - h - \lambda_2 - \lambda_3 = 0
\end{align*}
\]

The solution is

\[
\begin{align*}
\lambda_1 &= \lambda_2 \gamma + c_2 - c_1 \\
p_2 &= h + \lambda_2 + \lambda_3 \\
y &= -\frac{A(p_1 - c_2)}{R(1 - \theta)} - \frac{A\lambda_1}{R(1 - \theta)} + \frac{A\lambda_3}{R(1 - \theta) \theta}
\end{align*}
\]

In (5), since \(c_2 > c_1\) and \(\lambda_2 \gamma \geq 0\), we have \(\lambda_1 > 0\) from the first equation; since \(y > 0\), \(\lambda_1 > 0\) and \(p_1 > c_2\), we have \(\lambda_3 > 0\) from the third equation; and finally, since \(\lambda_2 \geq 0\), we have \(p_2 > h\) from the second equation. Therefore, we only need to consider the following cases with respect to \(\lambda_2\).

(a). \(\lambda_1 > 0, \lambda_2 = 0, \lambda_3 > 0\). The solution reduces to

\[
\begin{align*}
A \left[ 1 - \frac{p_1 - p_2}{R(1 - \theta)} \right] &= x \\
\gamma x &\geq y \\
\frac{A(p_1 \theta - p_2)}{R(1 - \theta) \theta} &= y
\end{align*}
\]

which leads to \(\lambda_1 = c_2 - c_1 > 0, \lambda_2 = 0, \lambda_3 = 2p_1 \theta - c_1 \theta - p_2 > 0\), and

\[
\begin{align*}
x^* &= A \left[ 1 - \frac{p_1}{R} - \frac{c_1 \theta - h}{2R(1 - \theta)} \right] \\
y^* &= \frac{A(c_1 \theta - h)}{2R(1 - \theta) \theta} \\
p_2^* &= p_1 \theta - \frac{c_1 \theta - h}{2}
\end{align*}
\]

The condition \(\gamma x \geq y\) in (6) yields

\[
2(R - p_1)(1 - \theta) \gamma \theta \geq (1 + \gamma \theta)(c_1 \theta - h)
\]

Moreover, \(p_2^* \leq \theta p_1\) holds since \(\theta c_1 \geq h\). \(p_2^* \geq h\) is equivalent to \((2p_1 - c_1) \theta \geq h\). With \(\theta c_1 \geq h\) and \(p_1 > c\), we have \(\theta c_1 + h < 2\theta c_1 < 2\theta p_1\), and thus \((2p_1 - c_1) \theta \geq h\). Therefore, \(h \leq p_2^* \leq \theta p_1\) holds.

(b). \(\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0\). The solution reduces to

\[
\begin{align*}
A \left[ 1 - \frac{p_1 - p_2}{R(1 - \theta)} \right] &= x \\
\gamma x &= y \\
\frac{A(p_1 \theta - p_2)}{R(1 - \theta) \theta} &= y
\end{align*}
\]

The last two equations in (8) yields \(p_2\), which leads to

\[
\begin{align*}
x^* &= A(R - p_1) (1 + \gamma \theta) R \\
y^* &= \frac{\gamma A(R - p_1)}{(1 + \gamma \theta) R} \\
p_2^* &= p_1 \theta - \frac{(R - p_1)(1 - \theta) \gamma \theta}{1 + \gamma \theta}
\end{align*}
\]

As a result, the solution in (9) corresponds to the condition

\[
2(R - p_1)(1 - \theta) \gamma \theta < (1 + \gamma \theta)(c_1 \theta - h)
\]

Moreover, \(p_2^* \leq \theta p_1\) holds since \(p_1 \leq R\) and \(0 \leq \theta \leq 1\). As for \(p_2^* \geq h\), note that

\[
2(R - p_1)(1 - \theta) \gamma \theta < (1 + \gamma \theta)(c_1 \theta - h)
\]

is equivalent to

\[
\frac{(R - p_1)(1 - \theta) \gamma \theta}{1 + \gamma \theta} < \frac{c_1 \theta - h}{2}
\]

which leads to

\[
p_2^* = p_1 \theta - \frac{(R - p_1)(1 - \theta) \gamma \theta}{1 + \gamma \theta} > p_1 \theta - \frac{c_1 \theta - h}{2}
\]

With an argument similar to the analysis in (a), we have \(p_2^* \geq h\). Therefore, \(h \leq p_2^* \leq \theta p_1\) holds.
3 Results

In accordance with the previous analysis, we can have the following main theorem with respect to different parameter settings.

Theorem 3.1

(a) If

\[2(R - p_1)(1 - \theta) \gamma \theta \geq (1 + \gamma \theta)(c_1 \theta - h)\]

the manufacturer’s optimal policy is

\[
\begin{align*}
  x^* &= A \left[ 1 - \frac{p_1}{R} - \frac{c_1 \theta - h}{2R(1 - \theta)} \right] \\
  y^* &= A(c_1 \theta - h) \\
  p_2^* &= p_1 \theta - \frac{c_1 \theta - h}{2}
\end{align*}
\]

(b) If

\[2(R - p_1)(1 - \theta) \gamma \theta < (1 + \gamma \theta)(c_1 \theta - h)\]

the manufacturer’s optimal policy is

\[
\begin{align*}
  x^* &= \frac{A(R - p_1)}{(1 + \gamma \theta)R} \\
  y^* &= \frac{\gamma A(R - p_1)}{(1 + \gamma \theta)R} \\
  p_2^* &= p_1 \theta - \frac{(R - p_1)(1 - \theta)\gamma \theta}{1 + \gamma \theta}
\end{align*}
\]

Proof. The proof can be derived from the previous analysis.

Theorem 3.1 concludes that the manufacturer’s optimal strategy is to set the right price of remanufactured products and the right supply quantities of new and remanufactured products such that all demands for new products are satisfied in the first period and, in the second period, only the remanufactured products are produced to satisfy all demands for remanufactured products.

Besides, by Theorem 3.1, we can have the following corollary on the relation between the optimal policy and return rate \(\gamma\).

Corollary 3.1

(a) When

\[2(R - p_1)(1 - \theta) \leq c_1 \theta - h\]

or when

\[2(R - p_1)(1 - \theta) > c_1 \theta - h\]

and the return rate \(\gamma\) satisfies

\[\gamma < \frac{c_1 \theta - h}{\theta[2(R - p_1)(1 - \theta) - (c_1 \theta - h)]}\]

\(x^*\) and \(p_2^*\) are monotonically decreasing as \(\gamma\) increases, while \(y^*\) is monotonically increasing as \(\gamma\) increases.

(b) When

\[2(R - p_1)(1 - \theta) > c_1 \theta - h\]

and the return rate \(\gamma\) satisfies

\[\gamma \geq \frac{c_1 \theta - h}{\theta[2(R - p_1)(1 - \theta) - (c_1 \theta - h)]}\]

the amount of available returned products for the remanufacturing process is always greater than the demand of remanufactured products, and thus \(x^*\), \(y^*\) and \(p_2^*\) are not affected by \(\gamma\).

Proof. When

\[2(R - p_1)(1 - \theta) \leq c_1 \theta - h\]

since \(c_1 \theta > h\), it is straightforward to derive

\[2(R - p_1)(1 - \theta) \gamma \theta < (1 + \gamma \theta)(c_1 \theta - h)\]

When

\[2(R - p_1)(1 - \theta) > c_1 \theta - h\]

and the return rate satisfies

\[\gamma \leq \frac{c_1 \theta - h}{\theta[2(R - p_1)(1 - \theta) - (c_1 \theta - h)]}\]

it is equivalent to

\[2(R - p_1)(1 - \theta) \gamma \theta < (1 + \gamma \theta)(c_1 \theta - h)\]

By Theorem 3.1, both conditions suggest that the manufacturer’s optimal policy is

\[
\begin{align*}
  x^* &= \frac{A(R - p_1)}{(1 + \gamma \theta)R} \\
  y^* &= \frac{\gamma A(R - p_1)}{(1 + \gamma \theta)R} \\
  p_2^* &= p_1 \theta - \frac{(R - p_1)(1 - \theta)\gamma \theta}{1 + \gamma \theta}
\end{align*}
\]

Obviously, \(x^*\) is monotonically decreasing as \(\gamma\) increases. As for \(y^*\) and \(p_2^*\), it can be observed that the key term is

\[\frac{\gamma}{1 + \gamma \theta}\]

whose derivative with respect to \(\gamma\) is

\[\left(\frac{\gamma}{1 + \gamma \theta}\right)' = \frac{1}{(1 + \gamma \theta)^2} > 0\]

Therefore, \(y^*\) is monotonically increasing as \(\gamma\) increases, while \(p_2^*\) monotonically decreasing as \(\gamma\) increases.

Otherwise, the condition

\[2(R - p_1)(1 - \theta) \gamma \theta \geq (1 + \gamma \theta)(c_1 \theta - h)\]

holds, and by Theorem 3.1, the manufacturer’s optimal policy is

\[
\begin{align*}
  x^* &= A \left[ 1 - \frac{p_1}{R} - \frac{c_1 \theta - h}{2R(1 - \theta)} \right] \\
  y^* &= \frac{A(c_1 \theta - h)}{2R(1 - \theta)\theta} \\
  p_2^* &= p_1 \theta - \frac{c_1 \theta - h}{2}
\end{align*}
\]
Apparently, $x^*$, $y^*$ and $p_r^*$ are not affected by $\gamma$. □

Corollary 3.1 indicates that when the return rate has effect on the manufacturer’s decision and increases, both the optimal quantity of new products and unit selling price of remanufactured products will decrease, while the optimal quantity of remanufactured products will increase, which makes sense from the economic viewpoint. Under the condition

$$2(R - p_1)(1 - \theta) > c_1 \theta - h$$

the return rate will no longer affect the manufacturer’s optimal policy when it exceeds a threshold.

4 Conclusion

This paper considers a two-period model in which the competition stems from two types of products with different qualities and prices. We derive the manufacturer’s optimal strategies with respect to various parameters’ settings, and investigate the impact of the return rate on the optimal strategies.

We currently consider the return rate, $\gamma$, as an exogenous parameter. Nevertheless, it seems more practical to treat $\gamma$ as a function of price and cost. In addition, it is interesting to relax the fixed market size, $A$, into a stochastic variable.

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