ROBUST P-MEDIAN MODEL FOR FACILITY LOCATION PROBLEM BASED ON SCENARIO ANALYSIS IN EMERGENCY MANAGEMENT

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Keywords: Facility Location; Robust Decision; Scenario Analysis; Emergency Management

Abstract

Uncertainty of disaster districts is an important character in emergency response, which brings great challenge for facility location decision before disasters. Many kinds of resources are reserved in these facilities in order to satisfy the requirement of affected districts. Careful deployment of facilities can decrease the response time and improve the ability of emergency management. In this paper, scenario analysis is proposed for forecasting disaster districts under uncertainty. In order to make robust decision, a robust p-median model is presented for facility location problem in dealing with the specified percent of affected districts. Since the robust model is NP-hard, an approximation algorithm is designed and theoretical ratio is analyzed. Finally, sensitivity analysis and performance evaluation for the proposed model and algorithm are shown by computational instances.

1 Introduction

More and more natural and man-made disasters occurred at every corner of the world, especially in recent years. And scales of the disasters are so large that they brought much loss of lives and properties. Earthquakes, hurricanes, tsunamis, terrorism attacks, etc. are among the causes of large-scale disasters. September 11, 2001 in USA, SARS in 2003, hurricane Katrina in 2005, Wenchuan Earthquake in 2008, 2010 Haiti earthquake, 2011 Japan Earthquake, and Indian Ocean Tsunami in 2004 are all very destructive disasters.

In order to relieve the damage of large-scale disasters, many countries pay attention to emergency management issues. Mitigation and preparation are carefully considered in one hand, while quick and efficient response is done on the other hand. Then as a new multidisciplinary field, emergency management has caught more and more researchers in various fields including management science, computer science, operations research, psychology etc. Optimization methods are useful tools in many decision making processes. Altay (2006) [1] and Larson (2006) [2] discussed operations research in disaster response in details.

Facility location problem is one of the important research topics in emergency management. Many kinds of resources are reserved in these facilities in order to satisfy the requirement of affected districts. Careful deployment of facilities can decrease the response time and improve the ability of disaster relief operation. Meanwhile, it is difficult to forecast the affected districts before the disasters. Then, facilities need to be located for dealing with all possibility of disaster districts, which is a much more complex decision problem.

Usually, the coverage method is used for decision support by considering the whole region. In this situation, it assumes that all districts are destroyed by the disaster. Obviously, it is the worst case. However, this may not be practical. In many cases, the impact level of a disaster is a part of the region, such as flood, earthquake in a province, which represents the province as a region. For a specified region, the impact level of a disaster needs to be estimated before decision-making using statistical data. In this paper, percent of the region is used for describing the impact level.

100 percent is a special case of the impact level which represents the whole region is destroyed. For simplification, a region is represented as a node set and each node is a potential affected district. Another node set is used as candidates set for facility locations, from which decision-maker needs to select several in dealing with a specified impact level of disasters, and all possible scenarios in this level must be considered. A robust model is presented for this problem in this paper.

The remainder of this paper is organized as follows. Literature reviews are summarized firstly in section 2. In section 3, parameters description and robust model formulation are presented in detail. While an approximation algorithm is proposed and analyzed theoretically in section 4. In section 5, sensitivity analysis and performance evaluation for the proposed model and algorithm are shown by computational instances. Conclusions and future research problems are discussed in the section 6.

2 Literature Review

Facility Location Problem (FL) is a classical issue which has been studied since Weber Location Problem in 1909. The problem looks for the best locations for a set of facilities that must satisfy requests of service coming from a given set of customers. The ubiquity of locational deci-
tion-making has led to a strong interest in location analysis and modeling within the operations research and management science community. There are many facility location models, such as covering (\([3, 4]\)), p-center (\([5, 6]\)), p-dispersion (\([7]\)) and p-median (\([8]\)). Cooper \([8]\) formulated the facility location-allocation (FLA) that provides a valuable method in deciding where to place facilities coupled with determining how to assign demand to the located facilities in order to utilize resources effectively. To optimize transportation between facilities, Gupta et al. \([9]\) introduce Connected Facility Location Problem (ConFL) which corresponds to the above mentioned facilities. The facilities need to be installed and connected with each other and customer nodes need to be assigned to them. The ConFL problem consists of finding an assignment of each customer to exactly one facility and connecting these facilities via a Steiner tree. Furthermore, the ConFL is transformed into the minimum Steiner arborescence problem and solved by an exact branch-and-cut method. Ten different integer programming formulations for ConFL have been presented by Gollowitzer and Ljubić\([10]\). Several relevant special ConFLs are considered in some recent works, such as capacity constrained ConFL \([11, 12, 13]\), hop constrained ConFL \([14, 15]\).

Among the inventory management, customer demand may be assumed to be uncertain. Gülpinar \([16]\) considers a stochastic facility location problem and a stockout probabilistic requirement stated as a chance constraint. They study robust approximations to the problem in order to incorporate information about the random demand distribution in the best possible and computationally tractable way. Robust optimization strategies for facility location appear to have better worst-case performance than non-robust strategies. Wang \([17]\) focuses on the logistics center location and allocation problem under uncertain environment. A regret model is proposed to compare with stochastic model and deterministic model. Gabrel \([18]\) divides the location problem into two stages, before and after the realization of uncertain parameters. They proposed a 2-stage robust formulation for the location transportation problem. Lu \([19]\) pays attention to uncertainty of travel times between urgent relief distribution centers (URDCs) and affected areas. By using fixed intervals or ranges instead of probability distributions, they present robust vertex p-center model.

In this paper, facility location is considered as one stage decision problem while dealing with all possible scenarios under a specified impact level. A robust p-median model will be proposed in the next section.

### 3 Robust p-median Model

In this section, we will formulate the robust p-median model for facility location problem. Firstly, the impact level should be specified. Let \(I\) and \(J\) be the potential affected districts set and facility locations candidates set respectively. The number of facilities is \(p\). The impact level is represented as a percentage \(\alpha\), which means any subset \(V_s \subseteq I\) with \(|V_s| = \alpha |I|\) is one scenario \(s\). \(S\) is the set of scenarios. Then, the number of scenarios is combinatorial number \(C^{\left| I \right|}_{\left| S \right|}\). \(d_{ij}\) is the travel distance from \(i\) to \(j\), for any \(i \in I\) and \(j \in J\).

The decision variables are \(X_j\) and \(Y^s_{ij}\), for any \(j \in J\), \(i \in I\) and \(s \in S\).

\[
X_j = \begin{cases} 1 & \text{candidate } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}
\]

\[
Y^s_{ij} = \begin{cases} 1 & \text{affected district } i \text{ is serviced by facility } j \text{ under scenario } s \\ 0 & \text{otherwise} \end{cases}
\]

The total response distance under scenario \(s\) is defined as \(f_s\), which satisfy:

\[
f_s = \sum_{i \in I} \sum_{j \in J} d_{ij} Y^s_{ij}
\]

Then, decision objective is to minimize total response distance for the worst scenarios. The Robust p-median Model (RpM) is defined as follows:

\[
\min z = \max \left\{ f_s \right\}_{s \in S}
\]

\[
s.t. \quad \sum_{j \in J} X_j = p
\]

\[
Y^s_{ij} \leq X_j, \quad \forall i \in V_s, j \in J, s \in S
\]

\[
\sum_{j \in J} Y^s_{ij} = 1, \quad \forall i \in V_s, s \in S
\]

\[
X_j = \{0, 1\}, Y^s_{ij} = \{0, 1\}, \quad \forall i \in I, j \in J, s \in S
\]

Since the number of scenarios is \(C^{\left| I \right|}_{\left| S \right|}\), RpM is difficult to solve especially for large scale instances. An approximation algorithm will be proposed in the next section.

### 4 Approximation Algorithm and Theoretical Analysis

In this section, we will present a quite simple approximation algorithm with approximation ratio \(\beta/\alpha\), where \(\beta\) is the best approximation ratio to the classical p-median problem.

Now, the special case for \(V_s = I\) is considered. Then, RpM can be written as a Classical p-median Model (CpM):
\[
\min z' = \sum_{i=1}^{ \mid I \mid} \sum_{j=1}^{ \mid J \mid} d_{ij} Y_{ij}
\]

\[\text{s.t. } \sum_{j \in J} X_j = p \]

\[Y_{ij} \leq X_j, \quad \forall i \in I, j \in J\]

\[\sum_{i \in I} Y_{ij} = 1, \quad \forall i \in I\]

\[X_j = \{0,1\}, Y_{ij} = \{0,1\}, \quad \forall i \in I, j \in J\]

Next, the approximation algorithm based on CpM for RpM is designed as follows:

**Algorithm based on CpM (AbC)**

1. **Step 1**: Given an instance of RpM, construct CpM.
2. **Step 2**: Solve CpM by the best possible approximation algorithm, and suppose \(\{X^*, Y^*\}\) is the solution.
3. **Step 3**: Output \(X^*\) as the scheme of facility location.

Since CpM is also NP-complete, let \(\beta\) be the best approximation ratio. It is easy to prove the following theorem.

**Theorem 1.** Algorithm AbC can solve RpM in polynomial time with approximation ratio \(\beta/\alpha\).

### 5 Sensitivity Analysis and Computational Results

In this section, we will analyze the sensitivity of impact level \(\alpha\) and performance of RpM and algorithm AbC. Suppose there are \(|I|=40\) potential affected districts and \(|J|=20\) facility candidates randomly laying on a 400*400 plan as shown in Figure 1. The response distance \(d_{ij}\) is Euclidean distance between facility and district. It is required to select \(p=8\) locations from \(J\) to open facilities.

Figure 1. Instance

Figure 2. Solution of CpM

Firstly, for a specified impact level \(\alpha\), we will compare the approximation solution which can be solved by Matlab optimization tools for small instance, with optimal solution that can be enumerated. Figure 2 shows the optimal solution of CpM, which will be the approximation solution of RpM according to algorithm AbC.

Figure 3. Solution of RpM with \(\alpha = 0.4\)

Table 1. Computational results

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>app.</th>
<th>opt.</th>
<th>gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>380.2</td>
<td>350.7501</td>
<td>29.4499</td>
</tr>
<tr>
<td>0.2</td>
<td>688.5</td>
<td>657.4673</td>
<td>31.0327</td>
</tr>
<tr>
<td>0.3</td>
<td>969.7</td>
<td>939.0979</td>
<td>30.6021</td>
</tr>
<tr>
<td>0.4</td>
<td>1204.5</td>
<td>1193.2</td>
<td>11.3</td>
</tr>
<tr>
<td>0.5</td>
<td>1408.9</td>
<td>1408.9</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>1604.1</td>
<td>1604.1</td>
<td>0</td>
</tr>
</tbody>
</table>

The optimal solution for RpM with \(\alpha = 0.4\) is shown in Figure 3 as an example for explaining the difference against approximation solution output of algorithm AbC. From Table 1, we can get the following conclusions:

1. Algorithm AbC is sensitive to impact level \(\alpha\). The solution becomes closer to that of RpM when \(\alpha\) goes to 1.
2. The gap between approximation value and optimal value is getting much smaller when alpha approach to 1.
6 Conclusion

In this paper, a robust model for facility location problem is presented in dealing with uncertainty of emergency management. According to scenarios of affected districts, the robust p-median model is formulated and approximation algorithm is proposed. Finally, the sensitivity of impact level $\alpha$ and performance of model and algorithm are analyzed.

For future works, the model may be analyzed further according to all impact level with different occurring probability.

Acknowledges

This work was supported in part by the National Natural Science Foundation of China under Grant No. 71001099.

References