Dynamical complexity in a predator-prey eco-epidemical system

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***Abstract*—Effects of the relationship between species and environment on an eco-epidemiological system are investigated. And periodic variation is also added to the disharmony parameter. The dynamic behaviors of the system are simulated numerically. A variety of complex population dynamics including stable state, periodic resonance and chaos are obtained. The most important result is that harmony relationship between prey species and environment is benefit for the controlling of disease. Our result reinforces the conjecture that the relationship between species and environment is crucial to transmission of infectious disease.**

Keywords—eco-epidemiology; harmony; disharmony; periodic

I. INTRODUCTION

Eco-epidemiology is a new branch in mathematical biology which considers both the ecological and epidemiological issues simultaneously. Understanding the dynamics of a predator-prey system with disease is an important subject in the eco-epidemiology [1]. Epidemiological processes like disease may cause vital changes in the dynamics of an ecosystem. Recent papers have showed that complex dynamics of eco-epidemiological system may be induced from some key parameters, e.g., intrinsic growth rate, contact rate and carrying capacity of the environment [2]. Moreover, successful invasion of a parasite into a host or community can depend crucially on other factors, such as community structure, infection strength and also seasonal varying environment [3]. Do environmental factors or other system parameters produce periodic or chaotic dynamics? Therefore, a better understanding of complex dynamics of an infected prey-predator system is considerable interest.

In nature, the relationship between population and environment could qualitatively affect the dynamics of population communities [4]. The harmony relationship can increase implicitly the carrying capacity and then raise the growth rate of population. Moreover, population communities are usually imbedded in periodically varying environment, and the relation between population and environment may be impacted by the seasonal or periodic forcing [5, 6]. A variety of studies have been performed on the interactions between seasons and internal biological rhythms, and these interactions could induce limit cycle and chaotic behaviors [5].

Here, we will set up two eco-epidemiological models based on the ordinary differential equation to study the above issues. First, we consider an infected prey-predator model with the constant relationship between prey and environment. Through numerical simulations, we will present the dynamic behavior corresponding to different intensity of the harmony or disharmony strength. Second, using the above model, we will firstly discuss the effects of disharmony relationship parameter which influenced by periodic environment on the behaviors of the eco-epidemiological system.

II. MODEL

A. An eco-epidemiological model with constant disharmony relationship

The team of J. Chattopadhyay made notable progress in modeling eco-epidemiological dynamics. The original eco-epidemiological model from the team [7] is given by the system of three ordinary differential equations:

$$\begin{cases} \dfrac{dS}{dt} = rS\left(1-\dfrac{S+I}{K}\right) - \lambda SI - \alpha SP \\ \dfrac{dI}{dt} = \lambda SI - \beta IP - \mu I \\ \dfrac{dP}{dt} = -\varepsilon\beta IP - \delta P + \varepsilon\alpha SP \\ S(0) = S_0, I(0) = I_0, P(0) = P_0 \end{cases} \tag{1}$$

where S, I, P represent the densities of susceptible prey population, infected prey and predator population, respectively. The parameters have the following meanings: r, the maximum growth rate of susceptible prey; K, the carrying capacity; λ, the infection rate; α, β attacking rate on susceptible prey and infected prey respectively; μ, the death rate of infected prey; δ, predator death rate; ε, conversion efficiency.

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Mass action incidence is appropriate for a constant population, but standard incidence is more appropriate for a large population [8]. Thus, we will replace λSI by $\lambda SI/(S+I)$ in (1). To model the relationship between prey and environment, we proposed the power of a mathematical model, $\left(1-(S+I)/K\right)^D$, to describe the nonlinear relationship. Here, the exponent D is defined as disharmony relationship of prey species with environment. Then, we can obtain the following modified model:

$$\begin{cases} \dfrac{dS}{dt} = rS\left(1-\dfrac{S+I}{K}\right)^D - \lambda\dfrac{SI}{S+I} - \alpha SP \\ \dfrac{dI}{dt} = \lambda\dfrac{SI}{S+I} - \beta IP - \mu I \\ \dfrac{dP}{dt} = -\varepsilon\beta IP - \delta P + \varepsilon\alpha SP \\ S(0)=S_0, I(0)=I_0, P(0)=P_0 \end{cases} \quad (2)$$

When the power of the model (2), is larger than 1, means the way of activity (or living) of prey species is unfavorable to its future generations. Otherwise, $D<1$ indicates that prey populations improve the utilization ratio of the surplus capacity of habitat through favorable interaction with environment.

B. Relationship between species and environment affected by seasonal forcing

Moreover, to model the seasonal forcing, we assume that parameter D should be taken as periodic function of time, i.e., $D' = D(1+\sin wt)$. D' is periodical varying relationship and w is the angular velocities. It is not possible to deal with the seasonally forced system analytically, so we will treat them numerically below.

III. Analysis of the Model

We investigated the complex dynamics of the system (2) impacted by the disharmony parameter through numerical integration, a more intriguing behavior of the eco-epidemiological system was revealed. First, we found that two kinds of attractor depending on the disharmony constant D: the point attractor and limit cycle (Fig. 1). Second, we can obtain that Hopf bifurcation takes place at approximately $D_{cr} = 0.495$ (we can obtain it from simulations). When $0<D<D_{cr}$, the dynamics of system (2) may exhibit a limit cycle. But as $D>D_{cr}$, the system shows a stable behavior. Then, the relationship between prey species and environment can induce complex dynamics. Unstable dynamics could be occurred if the prey species can harmonize itself with environment.

Next, we try to analyze the systematic behavior of the susceptible prey, infected prey and predator populations with respect to the different values of the relationship exponent D (Fig. 2-4). Let us take the five different values of the parameter D viz. $D = 0, 0.35, 0.55, 1, 3$. Particularly, as the disharmony parameter $D=0$, which means the susceptible prey growing exponentially, the susceptible prey outbreak but least disappearance (Fig. 2a). Same type of phenomena occurs for the predator population (Fig. 4a), but infected prey population was extinct (Fig. 3a). The temporal dynamic behaviors of three populations also verified the above result from Fig. 1: as $D<D_{cr}$, the susceptible prey population is unstable (Fig. 2a, b), whereas as D crosses the critical value of D_{cr}, the susceptible prey population becomes stable and as the value of D tends close to D_{cr}, population takes long time to become stable (Fig. 2c). Same type of phenomena occurs for infected prey population (Fig. 3) and for predator population (Fig. 4).

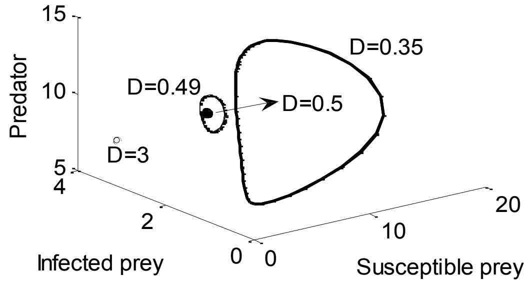

Fig. 1. The attractors of system (2) under different relationship between prey populations with environment. Parameters in Fig. 1-Fig. 5 are: $r=3, K=50, \lambda=0.5, \alpha=0.25, \beta=0.02, \mu=0.1, \varepsilon=0.2, \delta=0.15$, $S_0=10$, $I_0=1$, $P_0=5$.

Moreover, important phenomenon is observed here. From Fig. 3c, we notice that infected prey population boosts with increasing of the value D. Thus, we may conclude that the infectious disease can be controlled for the harmony relationship between prey population and environment. But for susceptible prey, the density changes slightly as increasing of parameter D (Fig. 2c). From Fig. 4c it can be easily deducted that when the parameter D increases then predator populations decline.

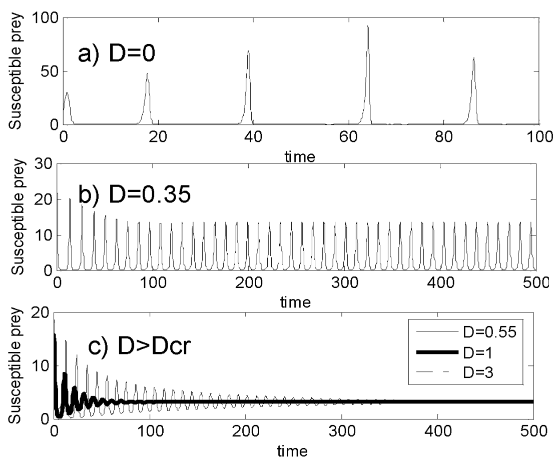

Fig. 2. $S-D$ plot in supercritical Hopf bifurcation for different values of D. The other parameters are the same as Fig.1.

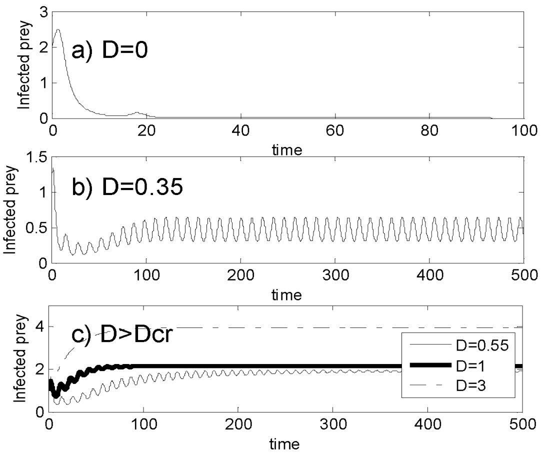

Fig. 3. $I-D$ plot in supercritical Hopf bifurcation for different values of D. The other parameters are the same as Fig.1.

Lastly, we consider the dynamic behaviors of the system where periodic forcing is superimposed on the disharmony parameter (Fig. 5). In order to mimic the annual cycle of environment, we here choose $\omega = 2\pi/365$. We notice that when $D = 0.5$, the relationship between prey population and environment is also harmony even through the system affected by seasonal forcing (D' changes from 0 to 1). But as $D = 1$, the relationship may change between harmony ($D' \in (0,1)$) and disharmony ($D' \in (1,2)$) due to seasonal forcing. It can be observed that adding seasonal forcing can lead to chaos (Fig. 5).

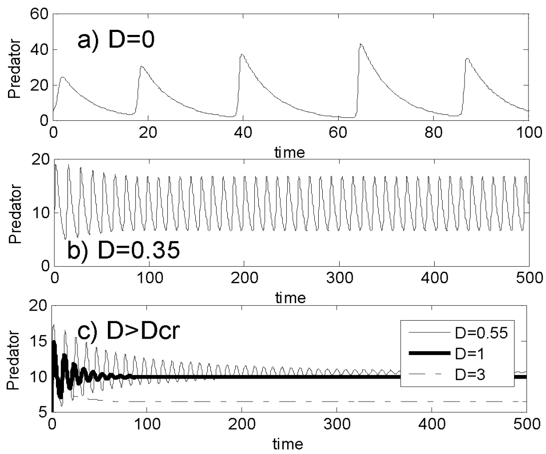

Fig. 4. $P-D$ plot in supercritical Hopf bifurcation for different values of D. The other parameters are the same as Fig.1.

IV. Conclusion and Discussion

We have investigated the dynamics of a simple eco-epidemiological model which consider the relationship between prey species and environment. Stable state, periodic oscillation and chaos behavior are obtained based on the above eco-epidemiological systems. From our simulation results, we draw the following major conclusions: (1) Hopf bifurcation is of direct relevance to the disharmony exponent between prey population and environment. (2) High harmony relationship can control the spreading of infectious disease in prey populations. The biological reasons could be due to as follows: harmony relationship between prey species and environment can boost its carrying capacity, and thus is benefit for the growth rate of susceptible prey populations. Then, the predator population may increase which profits from the raise of susceptible prey sizes. Due to the predator-prey cycle that occurs in Lotka-Volterra model [9], the decline of infected prey may be due to the rapid increase of predators. (3) Adding periodic forcing could lead to chaotic dynamics even through the original autonomous dynamical system is local stable.

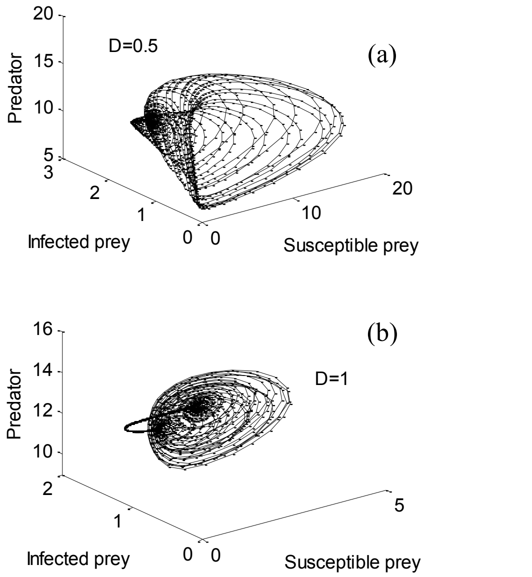

Fig. 5. The chaotic attractors of system (2) under two different relationship parameter D with periodic forcing: (a) $D = 0.5$; (b) $D = 1$. The parameter is $\omega = 2\pi/365$ and others are the same with Fig. 1.

The aim of mathematical epidemiology is possibly to obtain the dynamics of disease transmission and devise reasonable vaccination policies according to the spreading trend. Unlike some previous studies, we mainly interest in the influence of the relationship between prey species and environment, and also exogenous perturbations which represent environmental variations. Consulting the temporal dynamics of the infected prey populations, we know that the disease control could be possible when the relationship between prey populations with environment is harmony. From our conclusions, many phenomena found in this study could be directly related to infectious transmission. Due to seasonal perturbations, the dynamics of measles could be changed from damped oscillation to periodic oscillation, and further run to chaotic with increasing of the amplitude of seasonal forcing [10]. As such, all the results resented in this paper are only a starting point for understanding the spreading of eco-epidemiology, which surly requires further exploitations.

Acknowledgment

We are grateful to the editor and anonymous reviewers for their constructive comments. This work was supported by the

978-1-4799-1389-3/13/$31.00 ©2013 IEEE

National Natural Science Foundation of China (31000192) and China Postdoctoral Science Foundation (20100481159)

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